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AN

INTRODUCTION TO THE STUDY

OF

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AN

INTRODUCTION TO THE STUDY
OF
GENERAL PHYSICS

BY

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PREFACE

The Sixth Edition of "General Physics", Book I of late Prof. De's "Introduction to the Study of Physics" is now out in the market after being out of print for nearly one year. The revision of the book was taken up by late Prof. De himself and he died engaged in the work of revision. Thanks are due to Dr. Hrishikesh Rakshit for the pains he took in completing the work left unfinished by late Prof De.

The method in the treatment of the subject matter as adopted by late Prof. De has been maintained and the new additions and alterations have been effected in view of the present syllabus in Physics of the different Indian Universities.

Owing to new additions in the body of the book including insertion of new figures, the volume of the book has increased to a certain extent but the price has been kept unchanged. It may be noted here incidentally that as formerly, in the first portion of the book, an attempt has been made to give a systematic and clear exposition of the fundamental principles of Mechanics which are so necessary in the study of Physics.

Every attempt has been made to make the book up-to-date by inserting illustrations of the latest type and University questions of recent years.

While presenting the present edition of the book before the Professors and the students with the hope that it will be received with the same old cordial welcome, the Publisher begs to invite suggestions for further improvements in the book.

Calcutta
July, 1938

The Publisher.

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CHAPTER I

INTRODUCTION

1. Science—In our everyday life we come across various phenomena which require explanation to satisfy the inquisitive mind. The knowledge gained in our attempt to obtain these explanations is what we call **Science**. The word Science (derived from *scio*, to know) originally meant knowledge ; but it has now come to mean a *system of accurate and co-ordinated knowledge.*

It is however, very difficult to distinguish between common knowledge and scientific knowledge. Strictly speaking, all accurate knowledge is science. "Science and commonsense are," as Prof. Huxley observes, "not opposed, as people sometimes fancy them to be, but science is perfected common-sense."

Common knowledge is first obtained through the medium of our senses, viz., sight, smell, touch, taste and hearing. That knowledge must next be extended by careful *observation*, *experiment*, and *reasoning*. The methods of observation and experiment are again nothing new to us, for we are all constantly making observations every day in our life and making experiments upon one thing or another. A scientific observation differs from a common observation in being at

the same time full, precise and 'free from an unconscious inference.' It should be made with the assistance of accurate measurements, wherever possible. A scientific EXPERIMENT is likewise a careful observation performed by watching what happens when the known conditions of an event in nature are *artificially* produced, or when some of these antecedents are altered. By an accurate reasoning we put forward a hypothesis to state how some observed phenomena and some definite initial conditions are inter-related as effects and causes. A hypothesis which can satisfactorily explain a large number of experimental results becomes finally a Law of Nature.

Thus a Law of Nature is 'the expression of a definite connection between a cause and its effect.' For example, it is a law of nature that bodies, whenever unsupported, fall to the ground ; the cause is the existence of an attraction between the earth and the things on its surface. It states that from a certain set of circumstances a certain result always follows, —in other words,—*Nature repeats herself.* Thus science is in a position to foretell the future course of certain phenomena.

The knowledge of the Laws of Nature obtained as the result of scientific investigations constitutes Natural Philosophy or Natural Science.

2. Scope of Physics—The scope of Natural Science aiming at the study of the whole of Nature, *i.e.*, all the phenomena of the material world is, obviously, a very vast one. It has, however, been divided on general grounds into various branches each of which deals with a particular class of phenomena of the material world.

Thus the phenomena of the growth of animals and plants depending on vital forces have been grouped apart under the domains of *Zoology* and *Botany* respectively ; those involving the study of the nature

and movements of the heavenly bodies constitute the science of *Astronomy* ; the study of the minerals and that of the constitution of the earth's crust form the domains of *Mineralogy* and *Geology* respectively and so on.

Physics—in the widest sense of the term, (derived from *Physike*, natural) may be said to mean the science of the whole of nature and is thus identical with Natural Philosophy. In its present limited sense, however, it is taken to be a branch of the latter studying only those properties of *matter* which depend simply upon the states of bodies and *not* on their constitution. It also studies *energy* in its various forms.

Matter—is anything which occupies space and which is perceptible to our senses. It is the material or stuff which all bodies are made of. It possesses certain fundamental properties, *e.g.*, extension or magnitude, inertia, gravitation, etc., which shall be considered later on (Chap. XIII).

Energy—is the name given to a fundamental entity possessed by matter which enables it to do work. Thus the energy of a body or a system is its capacity to do work.

When a piece of matter undergoes a *physical change*, its form or state only is altered but its composition remains the same. For instance, ice, water and steam all have the same composition and hence when ice changes into water or water into steam they are said to undergo physical changes.

Those changes of matter which are concerned with the composition of bodies and the interaction of one kind of matter with another are called *chemical changes* and are studied in the Science of Chemistry. When a body undergoes a chemical change, it disappears giving rise to new substances with entirely different composition. When a piece of paper is

burnt the ash left behind is quite a different substance. Thus iron rust is not iron, gunpowder after explosion is no more gunpowder.

3. Subdivisions of Physics—The science of Physics is usually divided into the following branches :—

(i) **General Physics**—dealing with the general laws of motion of bodies and the properties of matter.

(ii) **Acoustics**—studying the cause, the propagation and the nature of Sound and the relation of tones in music.

(iii) **Heat**—studying the effects of application of heat on bodies and the different ways of transmission of heat from one body to another.

(iv) **Optics**—studying the phenomena of light.

(v) **Magnetism**—studying the properties of magnets.

(vi) **Electricity**—studying the properties of electric charges and currents.

4. Subdivisions of General Physics—In the present treatise we make an elementary study of the portion of *General Physics* which again may be divided into two parts :—

PART I.—MECHANICS.

PART II.—PROPERTIES OF MATTER.

The term *Mechanics* was originally used by Newton to designate the *Science of Machines and the Art of making them.* It is now, however, generally applied to mean the *Science which studies the action of forces on matter producing motion or rest.*

Mechanics can generally be divided into the following two branches :—

(1) **Statics**—which studies the action of forces producing equilibrium or rest of a body.

1 kilometre (km.) = 0·621 mile
1 yard (yd.) = 0·914 metre
1 foot (ft.) = 30·48 centimetres
1 inch (in.) = 2·54 centimetres
1 mile (mi.) = 1·6 kilometres

It is worth noting here that there are cases in which smaller units than a millimetre are more convenient. In such cases, decimal parts of 1 mm. are taken as follows :—

1 micron (μ) $= \frac{1}{1000}$ mm. $= 10^{-3}$ mm.

1 milli-micron ($\mu\mu$) $= 10^{-6}$ mm. $= 10^{-7}$ cm.

1 Angstrom ($\mathring{A}$ or A. U.) $= 10^{-10}$ metre

$= 10^{-8}$ cm. $= 10^{-1}$ $\mu\mu$.

1 X -unit (X.) $= 10^{-11}$ cm. $= 10^{-3}$ $\mathring{A}$.

On the other hand, astronomical distances, which are very large are often expressed in terms of the **Astronomical Unit** which is equal to the mean radius of earth's orbit : thus

1 astronomical unit = 92,900,000 miles.

Another suitable unit very largely used in astronomical calculations is the **Light Year** which is the distance traversed by light in one year : thus

1 light year = 5865,000,000,000 miles.

9. Unit of Mass—The Mass of a body is defined as the total quantity of matter contained in it. It does not depend on the volume or space which it occupies. Thus a piece of India-rubber when compressed has a different volume, but retains the same mass. The mass is altered only by changing the quantity of matter in the body—in other words,—when the body gains or loses matter.

As there are two sets of units of length, there are also two sets of units of mass. The standard unit of mass in England, *i.e.*, in the F.P.S. system, is the

Pound Avoirdupois (*lb*), which is the mass of a certain piece of platinum that is preserved, like the standard yard, at the office of the Board of Trade.

The chief multiples and sub-multiples of this unit are—

1 stone	= 14 lbs.	1 ounce (oz.)	$= \frac{1}{16}$ lb.
1 hundred-wt.	= 112 lbs.	1 grain (gr.)	$= \frac{1}{7000}$ lb.
1 ton	= 2240 lbs.		

The unit of mass in the C. G. S. system is the **Kilogramme**. It is defined to be the mass of a certain lump of platinum preserved in the Archives in Paris. This was originally intended by Borda to be the mass of 1 cubic decimetre (= 1000 cubic centimetres) of distilled water at 4°C. It does not fulfil this condition exactly but is sufficiently near for all practical purposes.

The unit of mass usually adopted for scientific purposes is the **Gramme** (or gram) which is one-thousandth part of the mass of Borda's lump of platinum, known as the **Standard Kilogramme**. For practical purposes, however, a gram may be taken to be the mass of 1 cubic centimetre of pure water at 4°C.

Multiples and sub-multiples of the gram are as follws :—

1 decagram	= 10 grams.	1 decigram	$= \frac{1}{10}$ gram.
1 hectogram	= 100 grams.	1 centigram	$= \frac{1}{100}$ gram.
1 kilogram	= 1000 grams.	1 milligram	$= \frac{1}{1000}$ gram.

The relation between the units of mass in the two systems are given below :—

1 kilogram (kg.)	= 2·2 pounds.
1 pound (lb)	= 453·6 grams.
1 gram (gm.)	= 15·43 grains.
1 grain (gr.)	= 0·06 gram.

10. Unit of Time—The standard unit of time is derived from the rotation of the earth on its axis.

which is revealed to us by the apparent motion of the sun across the sky by day and of the stars by night.

Each day, the sun rises in the East, moves higher up in the sky until at mid-day it attains the highest altitude, when it is said to be in the celestial meridian (*i.e.*, the vertical plane which passes North and South through the observer) ; it then sinks lower and finally sets in the West. The interval of time that elapses between two successive passages, called *transits*, of the sun across the meridian of any place is called an **Apparent solar day.**

By careful observations it is found that the length of this apparent solar day is not constant but varies throughout the year. If, however, we add together the lengths of all the apparent solar days in a year and divide the sum by the number of days in that year, we obtain an interval of time which is constant from year to year and is known as the **Mean solar day.** The time indicated by our clocks corresponds to this mean solar day, such that

$$\begin{aligned} 1 \text{ mean solar day} &= 24 \text{ hours (hrs.) of our clock,} \\ &= 1440 \text{ minutes (mins.)} \\ &= 86400 \text{ seconds (secs.)} \end{aligned}$$

The unit of time on both the C. G. S. and F. P. S. systems is the mean solar second which is $\frac{1}{86400}$th part of a mean solar day.

Like the sun, the stars also rise, travel across the sky and after crossing the meridian, set below the horizon. But, unlike the case of the sun, the interval between two successive transits of any particular star across the meridian of any place is constant throughout the year. This interval is called a **Sidereal day.** Sidereal clocks indicating sidereal time are in use in astronomical observatories.

The length of the sidereal day is about 4 minutes shorter than the mean solar day ; it is calculated to

be 23 hrs. 56 mins. 4·09 secs. of mean solar time. Fig. 1 explains why the solar days are longer than the sidereal days.

Let PQR denote the orbit of the earth round the sun S. The earth revolves about its own axis in the

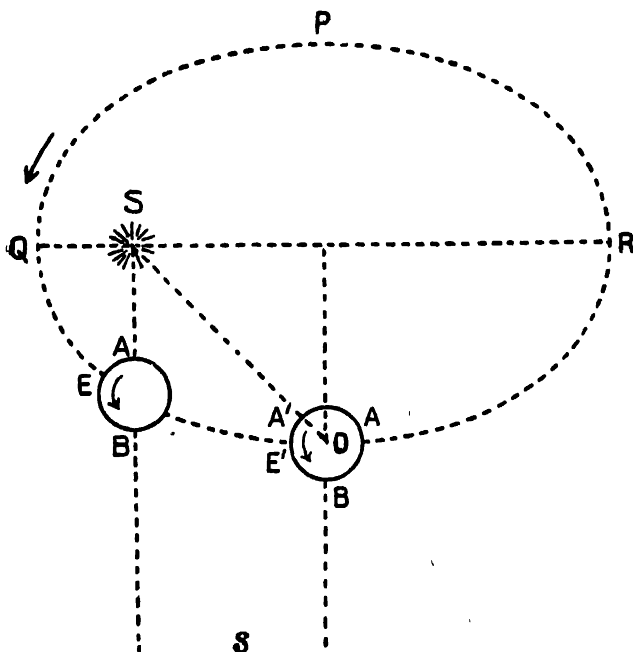


Fig. 1
Illustrating the difference between a solar and a sidereal day.

direction of the arrow as it advances in its orbit * in the direction PQR. Let E be the position of the earth when the meridian of the point B passes through a certain star S By the time the earth makes one

* The orbit is an ellipse with the sun at one of its foci.

complete revolution about its axis, it will reach the position E' in its orbit and the meridian of the same point B will again pass through **S**, for, the latter being very far away, the lines joining **S** and B in the two positions of the earth are parallel. Thus the sidereal day, which is the interval between two successive transits of a star across the meridian of a particular point, is equal to the period of revolution of the earth about its axis and is, therefore, constant. But the apparent solar day is longer than this. For, let the sun be in the meridian of the point A when the earth is at E. When, after completing one revolution, it comes to E′, the meridian through A will not reach S but will be behind it by the small angle AOA′. The apparent solar day is, therefore, longer than the sidereal day by the time required by the earth to rotate through this angle.

11. Dimensions of a Unit.—The value of a derived unit depends on the values of the fundamental units from which it is derived. The **dimensions of a unit** for measuring a physical quantity express how this is related to the three fundamental units of length $[L]$, mass $[M]$ and time $[T]$. For instance, the area of a rectangle is obtained by multiplying its length by its breadth, each being a certain number of the unit of length. Hence the dimensions of an area are given by $[L] \times [L]$ or $[L]^2$. Similarly, in measuring a volume the unit of length occurs to the third power for it is the product of length, breadth and depth, each of which is of the dimension of a length. So the dimensions of a volume is $[L]^3$. In expressing the dimensions of a certain quantity we are not concerned with the numerical factors or the particular values of the fundamental units involved.

Velocity is defined as the distance traversed in unit time ; hence its dimensions are $[\frac{L}{T}]$ or $[L]\,[T]^{-1}$.

Again the density of a substance is defined as its mass per unit volume so the dimensions of density are $[\frac{M}{L^3}]$ or $[M][L]^{-3}$

It is obvious that when two physical quantities or groups of quantities are equated, both sides of the equation must have the same dimensions. Thus a knowledge of dimensions is useful in checking whether an equation is correctly stated or not.

MEASUREMENTS

12. Limits of Accuracy—In every physical measurement there is a limit of accuracy beyond which the degree of precision cannot be relied upon. This limit depends upon the nature of the quantity being measured, the type of measuring instrument used and the mode of measurement adopted. Thus for measuring the mass of a lump of gold a sensitive balance is to be taken with much greater care and precision than when weighing a quantity of rice or coal. An error of an inch is quite negligible in measuring the length of a hall, while an error of one hundredth of an inch is a serious matter in the measurement of some parts of an automobile engine.

It is well to take an account of the percentage error when the measurement of a quantity is made with a view to examine whether the result is acceptable or not. Thus an error of ·1 mm in measuring a length of 2 mm amounts to an error of $\frac{\cdot 1}{2} \times 100$ or 5 per cent and may not be quite acceptable.

MEASUREMENT OF LENGTHS

13. Metre Scale—For finding the length or distance between any two points in physical measurements various appliances are used according to the accuracy desired. For all ordinary purposes, a metre-scale can be used. It is usually made of box-wood or steel and is about one metre in length.

It is generally graduated in inches and tenths of an inch along one edge or on one side, and in centimetres and millimetres along the other edge or on the other side.

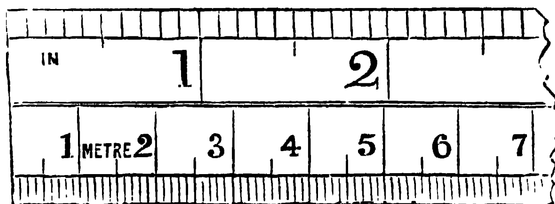


Fig. 2.
A metre-scale.

Such a scale is placed directly alongside the length to be measured (Fig. 2) and then the length is to be read off from the graduations of the scale. Where the direct application of the scale is not convenient a pair of ordinary dividers may first measure off the required distance and then be referred to a scale.

Simple Callipers—The simple callipers (Fig 3) consisting of two curved pieces of metal, hinged like a pair of scissors, are specially suited for measuring the diameters of pipes, hollow vessels, rods etc. One pair of legs, A, is meant for external while the other pair, B, for internal diameters.

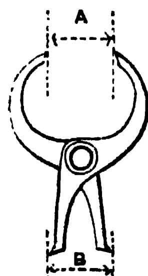


Fig. 3.

With a scale divided into millimeter or tenths of an inch it is possible with a little care and practice to measure approximately by "eye" a length up to one-fourth of a division ; hence the

accuracy obtainable with such a simple scale is very limited.

In cases where greater accuracy is wanted, the simple process of further subdivision of the scale does not help much. Other mechanical contrivances have to be adopted. Of these, the most commonly used are the *vernier* and the *micrometer screw*.

14. Vernier—The **vernier,** invented by Pierre Vernier, a French mathematician, is a simple and ingenious device for estimating lengths with greater accuracy than that obtainable by a simple scale. It is a short auxiliary scale, called the vernier, which slides along the main scale, whereby lengths are estimated to some particular fraction of the smallest division on the main scale, the zero line of the vernier acting as the index.

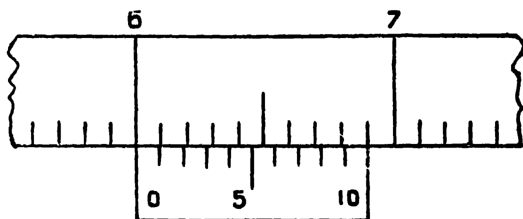


Fig. 4—Vernier.

In the form generally used, the vernier divisions are shorter than the main scale divisions. The vernier scale is so graduated that n divisions in it cover the same length as $n - 1$ divisions on the main scale. If v be the length of one vernier division and s that of one scale division, then we have

$$nv = (n - 1)s$$

Or $$v = s - \frac{1}{n}s$$

Or $$s - v = \frac{1}{n}s \quad \ldots \quad \ldots \quad (1)$$

Thus the difference between a scale division and a vernier division is $\frac{s}{n}$; this is called the **least count** of a vernier or the **vernier constant**

In fig. 4, the main scale is a centimetre scale graduated in millimetres. The vernier scale has 10 equal divisions equal in length to **9** scale divisions. Thus

$$10v = 9\ s$$
$$\therefore \quad 1v = \tfrac{9}{10}\ s$$

$\therefore$ Vernier constant = difference of 1 scale division and 1 vernier division.

$$= \left(1 - \tfrac{9}{10}\right) s$$
$$= \tfrac{1}{10}\ s = \cdot 1 \text{ mm.}$$

In fig. 5, AB is a rod whose length is to be measured. It is placed between the zero-end of the main scale on one side and that of the vernier on the other. It is seen that the zero line of the vernier lies between

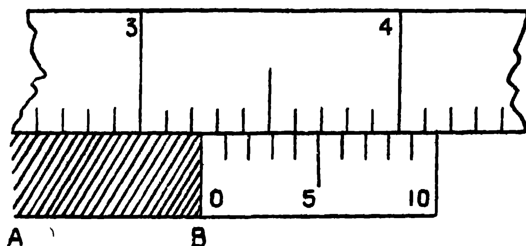


Fig 5.
Measurement of length with a Vernier

the marks 3·2 and 3·3 cm. on the main scale. Now looking along the vernier to see which division on it is most nearly opposite to a scale division it is seen that the fourth one is just opposite to the 3·6th mark on the scale. Hence :—

the 3rd. vernier division must be ·1mm. to the right of the 3·5th. mark on the scale,
the 2nd. vernier division must be ·2mm. to the right of the 3·4th. mark on the scale,
the 1st. vernier division must be ·4mm. to the right of the 3·3rd. mark on the scale,
and the zero line ·4mm. to the right of the mark 3·2 cms. on the scale. Hence the rod AB exceeds 3·2 cms. by ·4mm., *i.e.*, its length is 3·24 cms.

Thus to measure a length with the help of a vernier the reading on the main scale up to the division just before the zero line on the vernier is first noted. To this then is added the product of the least count and the number of the divisions of the vernier where coincidence occurs with an opposite scale-division.

The type of vernier described above is a *forward reading vernier* and is the kind generally used. In this, as is already seen, the vernier divisions are smaller than the scale divisions and the numbers on the vernier run the same way as the numbers on the main scale. There are also *backward reading verniers* in which n vernier divisions are equal to $(n+1)$ scale divisions so that a vernier division is larger than one scale division. These types have the numbers on the vernier run the opposite way to those on the main scale and are very rarely used.

It is to be observed that verniers are often constructed to give readings with greater accuracy than $\frac{1}{10}$ mm. On looking at eqn. (1) it may be seen that if n is increased, the value of the least count is reduced and hence increased accuracy is obtained. But one cannot thus push the accuracy to infinite limits. For as n is increased more and more, the error in ascertaining which vernier division exactly coincides with a main scale division is gradually increased thus putting a limit to the accuracy obtained.

The principle of the vernier is applied to a number of measuring instruments such as the *slide callipers, travelling microscope* and the *cathetometer.*

Slide Callipers—The slide Callipers is used specially for measuring the thickness of bodies and diameters of rods, balls etc. It consists of a steel scale and two steel jaws (Fig. 6). One of the jaws is fixed

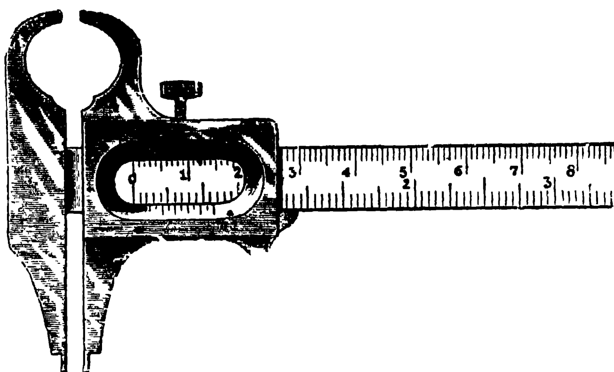


Fig 6
Slide-callipers

at one end of the scale at right angles to it while the other which is provided with a vernier and a fixing nut, is movable along the scale. When the two jaws are in contact, the zero of the vernier coincides with the zero of the scale. When the object to be measured is placed between the jaws so as to just touch them, its dimension is obtained in the usual way from the scale and the vernier.

Travelling or vernier microscope—Fig. 7 represents a **travelling microscope** or a vernier microscope as it is also called. It is a low power microscope mounted on a vertical stand and capable of being moved up and down along it. The stand, as a whole, can slide along a horizontal base by means of

a screw. The stand and the base are both provided with a fine scale. The microscope can be rotated to have its axis vertical or horizontal. The distance through which the microscope is moved either verti-

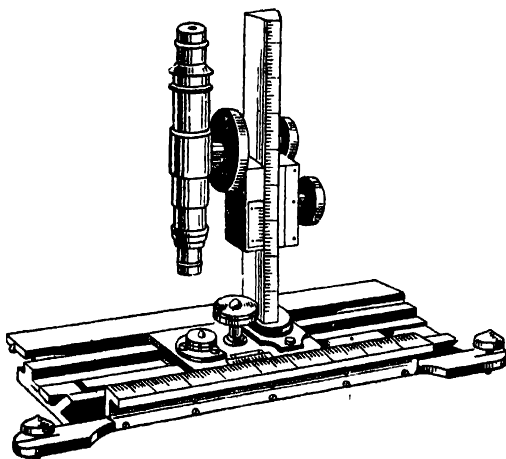


Fig. 7.
A travelling microscope

cally or horizontally can be read on the corresponding scale with the aid of a vernier which moves with the microscope or the stand. To measure the distance between two points, the microscope is first focussed on one point and then on the other. The difference between the two corresponding readings gives the distance required.

Cathetometer—The cathetometer is an instrument used to measure vertical heights. It consists of a vertical rod provided with a centimeter scale and an adjustable slide carrying a telescope with the axis horizontal. Thus the telescope can be moved up and down along the rod and its position can be

accurately read off on the scale with the help of a vernier carried by the slide. The method of measurment with this instrument is similar to that employed in the case of a travelling microscope.

15. Micrometer Screw—A form of apparatus by which short lengths such as the thickness of a plate or the diameter of a wire can be measured with great accuracy utilizes the application of a **micrometer screw.** It is an accurately cut screw working in a close-fitting nut. In one complete revolution the end of the screw advances through the nut by a distance equal to p Fig. 8(a), which is called the **pitch** of the screw, this being the distance between similar points on two consecutive threads of the screw. The screw is fitted with a circular head Fig. 8(b) of large diameter the edge of which is subdivided into a definite number of equal divisions, usually 50 or 100. When the head is rotated through a small fraction of a complete revolution, the end of the screw advances through the same fraction of its pitch. Thus very short lengths can be accurately measured.

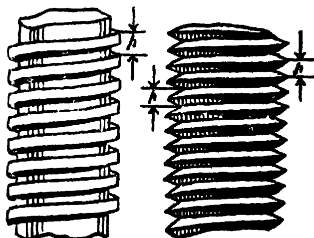


Fig. 8(a).

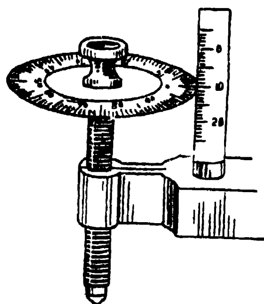


Fig. 8(b).

The accuracy obtainable by the use of a micro-

meter screw is limited by the accuracy of the pitch, *i.e.*, the accuracy with which the screw is cut and the closeness of fit of the screw and the nut.

The micrometer screw finds a practical application the *micrometer screw gauge*, the *spherometer* etc.

Micrometer Screw Gauge.—The micrometer screw gauge consists of a cylindrical tube S carrying a U-shaped arm F. A linear scale graduated in millimetres is engraved on S parallel to its axis. Inside the tube S moves an accurate screw, the pitch of which is usually ·5 mm. The screw is moved backward or forward by rotating the head H which also rotates the collar C. The bevelled end of C carries a circular scale (Fig. 9) of 50 or 100 equal divisions. The face B of the screw and the face A are exactly perpendicular to

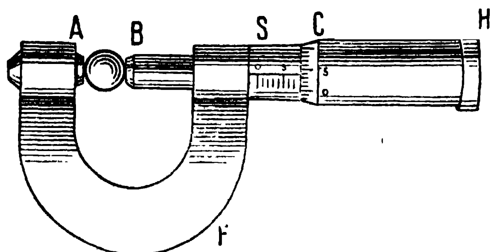


Fig. 9
A micrometer screw gauge

the axis of the screw. When these two faces are in contact, the zero of the circular scale coincides with the zero of the linear scale. The object to be measured is placed between the faces A and B so as to be lightly touched by them and the corresponding reading gives the required length.

In the common form of the screw gauge the pitch of the screw is ·5 mm. If the circular scale is graduated into 50 equal divisions, then the rotation of the screwhead through one circular division corresponds

to a movement of the screw end through $\cdot 5 \times \frac{1}{50}$ or 0·01 mm. The instrument thus reads to $\frac{1}{100}$ mm.

Spherometer.—The spherometer is used to determine the thickness of a plate and specially to measure the radius of curvature of a spherical surface, such as that of a mirror or a lens. It consists of a metal framework resting upon three fixed legs, A, B and C, pointed at the extremities (Fig. 10). The extremities are all in one plane and form an equilateral triangle. Through a nut at the centre of the frame works a micrometer screw, the axis of which is perpendicular to the plane of this triangle. The lower end of the screw forms the central leg of the instrument while the upper end is provided with a milled head M and a large graduated circular disc D. P is a vertical scale fixed at one end of the frame with its graduations close to those on the disc.

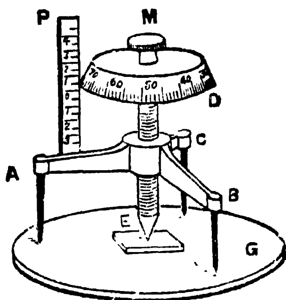


Fig 10
A spherometer

The pitch of the screw is usually ·5 mm. If the disc D carries 100 divisions, the least count of the instrument is $\cdot 5 \times \frac{1}{100}$ or ·005 mm.

If the spherometer be placed on an accurately plane surface, say a glass plate G, and the screw be turned down until its point just touches the surface, the zero on the graduated disc is opposite to the zero on the linear scale.

To get the thickness of a plate E, it is so placed on the plate G that the screw point just touches its upper surface, while the three outer legs still stand

on the glass plate. The corresponding reading gives the required thickness.

To determine the radius of curvature of a curved surface, say a concave mirror, the instrument is placed upon the curved surface and the screw is gently lowered till it just touches the surface.

The reading of the linear scale and the circular scale is taken. This gives the distance d of the screw point, Fig. 11 (a), below the plane of the feet ABC. The radius R of the surface is given by the relation

$$R = \frac{a^2}{6d} + \frac{d}{2} \qquad (2)$$

where a = distance between any two of the fixed feet.

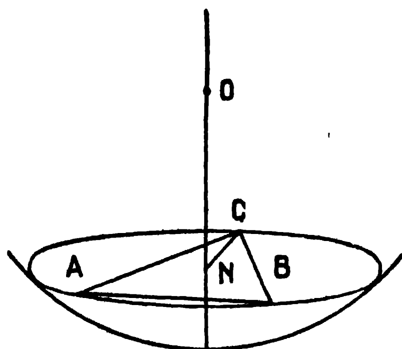


Fig. 11 (a)

This formula is obtained as follows :—

Let A, B and C be the points of contact of the fixed feet and D, that of the screw point on the curved surface. Let N be the position of the screw point in the plane of ABC and O the centre of curvature of

the spherical surface. Then ND $= d$ and AB $=$ BC $=$ CA $= a$ and OD $=$ R ; join CN. We then have

$$OC^2 = R^2 = ON^2 + NC^2, \text{ for } \angle ONC \text{ is a right angle}$$
$$= (R - d)^2 + NC^2$$

Now draw NE perpendicular to AC as in Fig. 11 (b). Then from geometry,

$$\frac{EC}{NC} = \text{Cos } \angle NCE = \text{Cos } 30^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore NC = \frac{2EC}{\sqrt{3}} = \frac{a}{\sqrt{3}}$$

$$\therefore R^2 = (R - d)^2 + \left(\frac{a}{\sqrt{3}}\right)^2$$

$$= R^2 - 2Rd + d^2 + \frac{a^2}{3}$$

$$\therefore R = \frac{d}{2} + \frac{a^2}{6d}$$

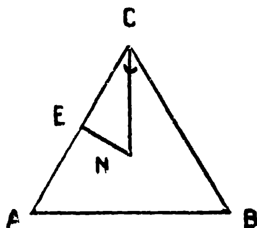


Fig. 11 (b).

MEASUREMENT OF MASS

16. Relation between Mass and Weight—It is necessary first of all to understand clearly the distinction between the mass and the weight of a body. From our daily experience we are familiar with the fact that all bodies on the earth possess *weight*, due to which they constantly tend to fall to the earth. The real cause of the weight of a body is the attraction exerted between the earth and the body, due to which the earth, having a much larger mass, pulls the body towards it. The centre of the earth is the point from which its attraction may be regarded to be exerted.

The **weight** of a body is thus the result of the earth's attractive pull exerted on the body. Now the magnitude of this pull, and hence the weight of a

body depend upon the mass of the body and upon its distance from the centre of the earth (art.58). *At a given place*, therefore, *the weight of a body is proportional to its mass*; in other words, bodies which are equal in weight, are also equal in mass. On this basis masses are measured or *compared* by the process of weighing. Indeed, the *measurement* of mass by weight is so general that the two words are commonly used as synonymous.

But the mass and the weight are two entirely different quantities. The mass of a body is the amount of matter it contains whereas its weight is the force of attraction exerted on it by the earth. While the mass of a body remains constant unless matter be added to or taken away from it, its weight may *vary from place to place* as its distance from the centre of the earth changes. For example, the weight of a body is found to increase slightly when it is taken from equatorial regions to polar regions. Again a body is observed to lose its weight, though very slightly, when it is taken to a considerable height above the sea-level, as on a mountain top or in a baloon ascent.

17. Balance—It has already been mentioned that as the weight of a body is directly proportional to its mass, any two masses may be compared by the process of weighing. The instrument used for this purpose is the **balance** by which the weight of the given mass is balanced against the weight of one or more standard masses.

The ordinary balance (Fig. 12) consists of a **horizontal beam** (1 in Fig. 12) balanced on a **knife-edge** of a triangular prism fixed at its middle. The knife-edge which is made of steel or agate to minimise friction, rests upon a plate of steel or agate fixed on the top of the supporting **pillar.** At the extremeties of the beam there are two knife-edges turned upwards to support

the inverted **stirrups** (2), from which the scalepans (3) are suspended. Hence the arms of the beams

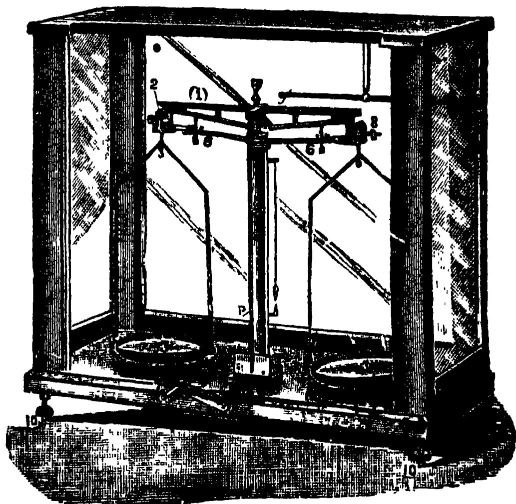


Fig. 12.
Balance.

measured from the fine knife-edge at the centre to those at either end are equal. A long pointer P, which is attached, at its upper part, to the centre of the beam, oscillates with it; when the beam is horizontal, the lower end of the pointer points to the zero mark of a graduated arc fixed on the pillar.

To preserve the sharpness of the knife-edges, the beam, when not in use, rests upon the **arresting-arrangement,** the under surfaces of the pans just touching the base-board at the same time. To secure this, the central pillar which supports the beam can be lowered by means of a small key or an eccentric arrangement fixed at the base of the pillar and

worked by a small handle (7) shown at the front of the base-board.

The base-board is provided with levelling screws.

At each end of the beam is fitted a screw (8) along which a small nut may be moved in or out whereby the effective weight of each arm can be altered through a small range.

To use the balance, this is levelled first of all by adjusting the levelling screw. If, when the beam is raised, the pointer does not swing through the same distance on either side of its zero position, as engraved on the scale below, either of the screw (8) is to be adjusted until this is secured, the balance is then ready for use.

The body to be weighed is placed on one pan and the standard weights, *i.e.*, the multiples and the sub-multiples of the unit mass are placed on the other pan until, on raising the beam, it is found to be exactly horizontal, *i.e.*, the pointer reads zero. From the principle of levers (art. 82), it follows that as the arms are equal, the downward forces on the two pans are equal; and as the downward force *i.e.*, the weight, is proportional to the mass, we can say that the corresponding masses are also equal,

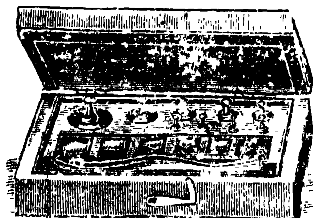


Fig. 13.
a weight-box.

The masses which are used as **standard weights**

are contained in a weight-box (Fig. 13). The weights are arranged within the box in an order, each being placed in a separate groove. The gram and its multiples are usually made of brass, while the fractional weights are of aluminium. The order of arrangement of the weights in an ordinary box is usually as follows—

(1) Brass weights,—

100, 50, 20, 20, 10 } gms.
5, 2, 2, 1 }

(2) Fractional weights,—

	0·5,	·2,	·2,	·1	*gm.*
marked	500,	200,	200,	100	*mgms.*
	·05,	·02,	·02,	·01	*gm.*
marked	50,	20,	20,	10	*mgs.*

These fractional weights fit into separate compartments and are all covered with a thick glass slab. On one side, a pair of forceps is also provided to handle the weights.

Weights smaller than 1 centigram are not very convenient to use. To avoid the use of such weights, the beam of a sensitive balance is often graduated into one tenths of the length of an arm, the zero being at the centre of the beam and the tenth divisions at the positions of the knife edges at the ends. A piece of bent wire, called a **rider,** 1 centigram in weight, can be moved along the beam by means of a rod projecting through a side of the balance case. When exact balance is obtained with the help of the rider, its position on the beam is read. The effect of placing the rider at a particular division, say S on one arm of the beam is equivalent to a weight of S milligrams placed on the corresponding scale pan.

Spring-balance—The spring-balance is another kind of instrument which is sometimes used for a quick and approximate measurement of masses. The mass of a

body is determined by observing the amount of deformation produced in a spring by the earth's pull exerted on it. One form of the spring-balance represented in fig. 14, consists of a spiral spring, the upper end of which is secured to the top of the semi-cylindrical case and the lower end is attached to a straight rod. The rod carries an *index* or a pointer which moves along a scale and has a hook attached to its bottom. When an object is hung from the hook, the spring is stretched, until its elastic forces balance the weight of the body. The scale is previously calibrated by hanging standard weights from the hook and marking their values opposite the positions of the pointer along the scale. The space between two such marks is subdivided into equal divisions, for the extension of the spring is found to be quite regular.

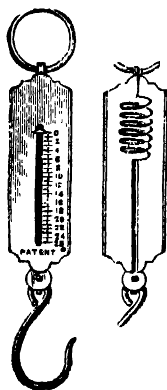


Fig. 14.
A spring-balance.

As the weight of a body changes, though very slightly, at different places on the earth, the pull on the spring also changes for the same mass. Strictly speaking, a spring balance gives accurate readings only when used at the place at which it was calibrated.

MEASUREMENT OF TIME.

18. Appliances for Measuring Time.—Any contrivance suitable for measuring time depends on some process which repeats itself in regular succession and in the same manner.

Hour-glass.—The hour-glass is a device used in the medieval ages to measure short lengths of time. It consists of two glass bulbs, each of about the same

size, joined by a narrow opening through which sand, water or mercury may run from one bulb to the other. The time taken for this passage serves as a standard length of time.

Sun-dial.—Perhaps the earliest method of reckoning the hours of the day was by means of the **sun-dial.** A flat metal disc, with the hours of the day marked upon it in a certain order on a circular dial, has a large pin or **style** as it is called, standing at the centre.

The movement of the shadow of the style with the progress of the sun along its apparent path in the sky indicates the hour of the day on the dial. The Greeks as well as the Hindus knew its use. The sun-dials at the observatories of Benares and Jaipur erected by Hindu kings are still worth seeing.

Towards the latter part of the sixteenth century (in 1584) GALILEO discovered the laws of oscillation of a pendulum (art. 93). He discovered that a pendulum which is simply a mass hanging at the end of a string will always take the same time to swing backwards and forwards, so long as the string is of the same length. In 1658, HUYGHENS first applied the pendulum to regulate the motion of clocks.

Clock.—The time-keeping instruments now generally used are **clocks** and **watches.** The mechanism in the clock is regulated in its motion by the oscillations of a pendulum. The main spring of a clock tends to rotate a toothed wheel D, called the **escapement wheel** (Fig. 15). A cross-bar E known as the **anchor,** is attached to the pendulum rod through the **crutch** B and the **fork** F and swings to and fro with it. Two projections on the cross-bar (M and N), called **pallets** engage alternately in the notches of the escapement wheel. Thus the motion of this wheel is arrested at the end of each swing of the pendulum.

The regulated rotation of the wheel is communicated through a set of wheels to the two hands of the clock which move over the dial. The escapement arrangement serves also to supply sufficient impulse to the vibrating pendulum at the right moment, as otherwise it would gradually come to rest on account of friction with the air.

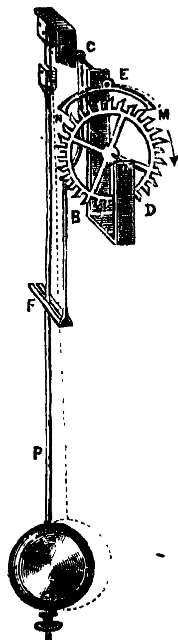


Fig. 15
The escapement of a clock pendulum.

In a watch time is measured by the oscillations of a fine elastic spring, furnished with a small fly-wheel, called the balance wheel, which is attached to an escapement arrangement. **A Chronometer** is an accurately constructed watch that keeps time with perfect regularity. Another variety of watch is the **stop-watch** which has a large 'second' hand revolving over the main dial once in a minute and a smaller 'minute' hand revolving over a smaller dial once in 30 minutes. The watch is provided with a spring stud at the top, by pressing which the 'second hand' is started. It may then be stopped, when necessary, by pressing the stud a second time. On pressing the stud a third time, the second hand flies back to the zero position. An ordinary stop-watch generally measures time upto $\frac{1}{5}$th. of a second. It is very convenient to be used where a small interval of time is to be noted.

Another instrument, called the **metronome** (Fig. 16) is much used to mark the time in practising music. It is virtually a pendulum provided with ticking

mechanism and is so constructed that its time of oscillation varies within certain limits. In fig. 16, the bob of the pendulum is just behind the wooden cover in front ; a small weight slides along the rod of the pendulum and can be fixed in any position. If the sliding weight is moved up, the pendulum swings slower ; and when it is moved down, the oscillations become quicker. Behind the sliding weight is a scale on which is written the number of ticks which the pendulum will make in a minute when the sliding weight is at a particular height along the rod. For example, if the weight be at the number 150, it means that 150 ticks would follow one another in 1 minute : hence the interval between two consecutive ticks is $\frac{60}{150}$ or 0·4 of a second.

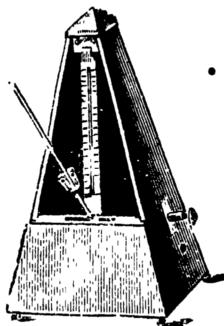


Fig 16
A Metronome.

MEASUREMENT OF SIMPLE DERIVED UNITS.

19. Measurement of Area.—For measuring an area, two lengths are to be measured and multiplied ; thus the area of a rectangle is given by the product of its length and breadth. Hence an area has two dimensions $[L]^2$.

Units of Area—The unit of area is a square of which each side is of unit length. In the C. G. S. system the unit of surface is a *square centimetre (sq. cm.)*. The British unit of area in physical measurements is a *square foot*.

For multiples and submultiples, we have

1 sq. cm. = 10 mm. × 10 mm. = 100 sq. mm.
1 sq. metre = 100 cm. × 100 cm. = 10,000 sq. cms.

1 sq. yd.	= 3ft.	×	3 ft.	=	9 sq. ft.
1 sq. ft.	= 12 in.	×	12 in.	=	144 sq. in.

The numerical relation between a sq. cm. and a sq. in. is shown below :—

1 sq. cm. = 0·155 sq. inch.
1 sq. in. = 6·45 sq. cm.

It is convenient to remember that 31 square inches are almost exactly equal to 200 square centimetres.

Area of Regular Plane Figures.—This can be readily calculated by the application of geometrical formulae on the measurements of certain lengths characteristic of the particular figure. Thus

Area of Rectangle	= length × breadth.
,, ,, Parallelogram	= base × altitude.
Area of Triangle	= $\frac{1}{2}$ × base × altitude
,, ,, Circle	= $\pi \times (\text{radius})^2$; $\pi = 3{\cdot}1416$.
,, ,, Ellipse	= $\pi \times$ semi-major axis × semi-minor axis.
,, ,, surface of a sphere	= $4\pi \times (\text{radius})^2$
,, ,, Curved Surface of a cylinder	= 2π × radius × length.

Area of Irregular Plane Figures.—The area of an irregular surface may be experimentally determined by placing the figure on a sheet of cardboard or a thin metallic foil of uniform thickness, and then cutting it out and weighing it. The weight of a known area of the same card or foil is then measured. From the comparison of the two weights the area of the figure is calculated.

Another method is to transfer the figure to a piece of squared paper and to count the number of small squares included within the figure, an approximation being made for the area represented by the incomplete squares on the boundary of the figure.

The product of this number and the known area of a small square determines the area of the figure.

20. Measurement of Volume.—The space occupied by a body measures its volume. To measure a volume, three lengths, *viz.*, length, breadth and height or thickness are required. Hence a volume has three dimensions $[L]^3$

Units of Volume—The unit of volume for all measurements is the volume of a cube, each edge of which is of unit length. In the C. G. S. system the unit is the *cubic centimetre (c.c.)*. For commercial purposes, the unit of volume is the LITRE, which is 1000 *c.c.* Large volumes of a liquid or a gas are generally expressed in litres.

The British unit of volume is the *cubic foot*. The unit in which the volume of a liquid is measured is a *pint* such that

1 gallon = 4 quarters = 8 points
1 pint = 20 ounces.

The numeral relations between the units is given below :—

1 c.c.	= 0·061 cu. in.	1 cu. in.	= 16·39 c.c.
1 litre	= 1·762 pints	1 gallon	= 4·541 litres.

Volume of Regular Solids, –

Volume of Parallelopiped = length × breadth × height.
,, ,, Cylinder = area of base × height.
$= \pi \times (\text{radius})^2 \times$ height.

Volume of Pyramid
or Cone $= \frac{1}{3} \times$ area of base × height.
,, ,, Sphere $= \frac{4}{3} \times \pi \times (\text{radius})^3$

Volume of Irregular Solids.—

The volume of a solid body, regular or irregular, may be easily obtained by the method of *displacement* of water. The solid is tied with a string and gently

lowered into a tall jar, provided with a spout at the side and filled with water. The water displaced by it raises the level of water, and the surplus flowing off by the spout is collected in a graduated cylinder G (Fig. 17), which measures the volume.

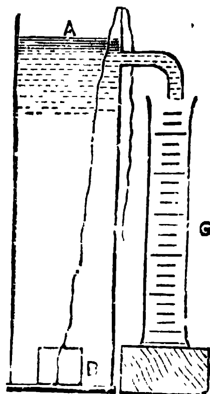


Fig. 17.
Volume by displacement of water

If the solid be small, it may be directly introduced into water contained in a graduated vessel. The difference between the two positions of the meniscus before and after the introduction of the body gives the required volume.

A more accurate method of finding the volume of a solid is by the application of the *Principle of Archimedes* (§144).

21. Measurement of Angle :—The ordinary unit adopted for measuring angles is the **degree.** A degree is $\frac{1}{90}$th. part of a right angle Hence 360° correspond to a complete rotation. A degree is divided into 60 minutes and each minute into 60 seconds.

Another unit of angle which is frequently used for theoretical purposes, is the **radian.** It is the angle subtended at the centre of a circle by an arc of circumference taken equal in length to the radius. When the radian is used as the unit, an angle is said to be measured in the *Circular Measure*. If θ is an angle subtended at the centre of a circle of radius r by an arc of length a, then $\theta = a/r$ radians.

Hence, the angle subtended at the centre by the circumference of a circle of radius r is $2\pi r/r$ or 2π radians.

Hence $\quad 2\pi$ radians $= 360^\circ$
Therefore $\quad 1$ radian $= 360^\circ/2\pi = 57^\circ{\cdot}2958$

An angle being measured as a ratio of two lengths $\frac{[L]}{[L]}$, has no dimensions.

The magnitude of an angle is found in practice by means of a **protractor** (Fig. 18). A protractor con-

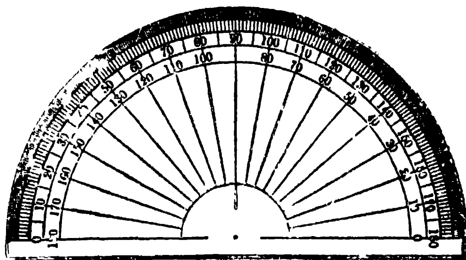


Fig. 18.
A protractor

sists of a semi-circular sheet along the circumference of which divisions are marked corresponding to the angles between the radii and the base of the protractor.

Exercise—I.

1. What are the three fundamental physical units and why are they so called ?

2. (*a*) How is the British unit of length defined ? Give its principal multiples and submultiples.

(*b*) What is a Metre ? Give the equivalents of the foot and inch in centimetres.

3. Explain the statement that we measure mass by weight.

4. Compare the British and French units of mass.

5. (*a*) What is a spring balance and what is its advantage over an ordinary balance ?

(*b*) A set of observations taken with a spring-balance is given thus :

Wt. in the pan (in grammes),—

10, 20, 30, 40, 50, 60, 70, 90,

Extension in mm,—

6, 13, 20, 24·5, 30·5, 33·5, 42·2, 55.

By means of a graph find the approximate relation between these two quantities. Find the magnitude of the weight that will extend the above spring by 40 mm. (C. U.—1914)

6. How will you find the volume of an irregular solid ? How do you check it ? (C. U.—1917)

7. Explain the meaning of '*an apparent solar day*' and '*a mean solar day*.'

8. Define mass, volume and density ; state the relation that exists between them.

What do you consider to be the best materials for the weights in an accurate box of weights ? Why is aluminium generally used for the fractions of a gram ? (Pat. U.—1918)

9. What is a radian ? What is the relation between a radian and a degree ?

PART II

MOTION



CHAPTER III

MOTION

22. Motion and Rest.—A body is said to be in *motion* when it changes its position ; thus a motion is *a change of position*. Conversely, if a body continues to occupy the same position for any length of time, it is said to be at rest.

A little consideration will show us that the state of rest possessed by a body can only be apparent , absolute rest, in other words, is unknown to us. A passenger in a railway carriage may be in a state of rest relative to the train in which he is travelling, but really he is in a state of motion with respect to the trees, houses etc., past which the train rushes. These trees, houses and other objects again are not in absolute rest, for they are sharing the motion of the earth which is moving round the sun with considerable velocity and is also rotating about its own axis. The sun itself has not only a motion of rotation, but is moving through space carrying with it the whole of the solar system of which it forms the centre. Even the stars are observed to have a motion, which excludes the idea of absolute rest.

Again, absolute motion is equally unknown to us. To measure an absolute motion a point absolutely fixed in space is first of all necessary, which, however is impossible to be realised by us.

Hence a body can only be in relative motion or relative rest according as it does or does not change its position with respect to its surrounding objects.

23. Motion of a body.—In mechanics, a body is supposed to be rigid, that is, its size and shape do not change. In practice, however, a perfectly rigid body cannot be realised. Still, the consideration of the dynamics of such a rigid body is useful as an introduction to the study of the actual complex cases.

Motion of a body may, in general, be divided into two kinds. A body is said to have a motion of **translation,** when it moves in such a way that the motion of all the particles of which we may consider the body to be built up, is exactly the same. Hence, any line within the body in its displaced state remains parallel to its original position. A train moving in a straight track, a boat sailing in a straight line, the fall of a stone in a well are examples of translatory motion.

On the other hand, when a body moves about a fixed point or axis, round which the particles within the body describe concentric circles, it is said to have a motion of **rotation.** Thus the motion of a grindstone on its axle, of a door on its hinges, of a pendulum about its point of suspension are instances of such motion.

But in actual cases, the motion of a body is frequently complex and is a combination of both rotation and translation. Instances of such motion are those of the wheel of a moving carriage, of a ball rolling along the ground, of a planet round the sun etc.

24. Displacement.—The displacement of a moving particle during a given time is its change of position in that time. It is measured by the straight line joining its initial position to its final position. The displacement posseses a direction as well as a magnitude.

When a body is undergoing a simple translation, the displacement is measured by the displacement of any particle in the body. In the case of simple rota-

tion, the displacement of a body is measured by the angular displacement of a particle in it.

25. Scalar and Vector Quantities.—In physics, any quantity may be classed into either of two types—scalar and vector. Any physical quantity which possesses magnitude only is a scalar quantity, no idea of direction is associated with it. Thus volume, mass, time, energy are all scalar quantities.

On the other hand, a physical quantity that possesses a direction in addition to its magnitude is called a vector quantity. For the full specification of such a quantity both magnitude and direction must be stated. Thus displacement is a vector quantity; it will be seen later that velocity, acceleration, force etc. are all examples of it.

A vector quantity can be represented by a straight line whose length is a measure of the magnitude and whose direction represents the direction of the vector, the sense of direction being usually indicated by small arrow-heads drawn either on the line or by its side.

Scalar Addition.—The net effect of the combination of two scalar quantities of the same kind is determined by ordinary arithmetical laws. Thus if 5 pounds of sugar be put into a bag containing 10 pounds of sugar, the net amount of sugar in the bag after this operation will be 15 pounds. Again, a man aged 30 years at present will, after 5 years, be 35 years of age. Similarly, if 1 gallon of petrol be used up from the reservoir of a car originally containing 3 gallons, the quantity of petrol left will be 2 gallons.

In all these cases, the ultimate result is obtained by ordinary arithmetical addition or subtraction. This is so because mass, time and volume are all scalar quantities.

Vector Addition.—But the combined effect of two vector quantities of the same kind cannot generally be found by direct arithmetical operations. The

single vector which will produce the same effect as two or more vectors of the same type jointly do, is called the resultant of these vectors. The process of finding this resultant is called vector addition or the composition of vectors. Vector addition is done by the method of *parallelogram of vectors* which may be stated as follows :—

If two vectors of the same type are represented in both magnitude and direction by two adjacent sides of a parallelogram, the diagonal of the parallelogram drawn through the point of intersection of these two sides represents the resultant vector in both magnitude and direction.

The propositions known as the *parallelogrm of velocities or forces*, which will be studied later, are particular cases of this general theorem.

26. Speed and Velocity.—The speed of a body is its rate of movement. This is measured by the change of position in unit time. It has no idea of direction associated with it ; it is a scalar quantity.

The **velocity** is the rate of change of position in a definite direction. It is thus a vector quantity and can be represented by a straight line.

Evidently velocity means speed in some definite direction. Thus a body moving in a curved line, say a circle, may have a constant speed but its velocity is not the same at all points, for its direction of motion is continually changing.

Uniform and Variable Velocities.—Speed and velocity may be either uniform or variable. A body is said to possess uniform speed when it passes over equal distances in equal intervals of time, *however small* these intervals may be. If, in addition, its direction of movement remains unaltered it is said to have uniform velocity. Thus when a body moves in a given direction so as to traverse 100 ft. in

every 4 second interval, its velocity will be said to be uniform only if its displacement is 25 ft. in 1 second, $12\frac{1}{2}$ ft. in $\frac{1}{2}$ second, 1 ft. in $\frac{1}{25}$th. of a second and so on.

When the velocity is uniform, it is measured by the distance traversed in a unit of time. If a particle passes over a space s in time t, the velocity v is given by the equation

$$v = \frac{s}{t} \qquad \dots \quad \dots \quad \dots \quad (3)$$

The unit of velocity is 1 cm. per sec. in the C. G. S. system and 1 ft. per sec. in the F. P. S. system.

The dimensions of velocity are $\frac{[L]}{[T]}$ or $[L][T]^{-1}$

From (3), we get,

$s = vt$. Or, distance traversed = velocity × time.

If a particle does not move over equal distances in equal intervals of time its speed is said to be variable. In such cases, the speed at any instant is measured by the rate at which the distance is being traversed during a small interval of time containing that instant. This interval of time is taken so small that the speed does not appreciably alter during this interval.

Thus, when we say that the speed of a railway train at certain instant is 15 miles per hour, we mean that had the train moved with a constant speed equal to that it actually possessed at the time under consideration, for a very short time, say half a second, it would have moved through 22 ft.*

When the speed of a body is variable, we may also define its motion by the average speed in any given interval. The average speed of a moving body in any given interval of time is the *uniform* speed with which

*30 miles per hour = 44 ft. per second.

it would have to move throughout that interval to traverse the same distance as that travelled in the actual motion. It is measured by the total distance travelled divided by the total interval of time during which the average speed is considered. If s is the distance traversed by a body moving in any manner in an interval of time t, then the average speed is given by $v = s/t$.

27. Acceleration—If the velocity of a body is not constant, it is said to have an **acceleration.** Acceleration of a body is the time-rate of change of its velocity, *i.e.*, the change of velocity per unit time.

As velocity has both magnitude and direction, acceleration implies an alteration of velocity either in magnitude, or in direction, or in both. As it has both magnitude and direction, it is a vector quantity and can be represented by a straight line.

If a body is moving at the rate of 10 ft. per sec. at any instant, and its velocity, one second later, is 15 ft. per sec., the velocity has obviously increased by 5 ft. per sec. in one second. The acceleration, therefore, is 5 ft. per sec. per sec.*

The unit of acceleration is 1 cm. per sec. per sec. in the C. G. S. system and 1 ft. per sec. per sec. in the F. P. S. system.

The dimensions of acceleration are

$$\frac{[L][T]^{-1}}{[T]} \text{ Or } [L][T]^{-2}$$

If the speed be increasing, *e.g.*, when an express train starts from a station, or a stone is let fall from

* The words *Per second* are repeated twice as the unit of time is involved twice in the measurement of acceleration. If the velocity of a railway train moving at the rate of 20 miles per hour increases to 25 miles per hour in one second, we may say that the acceleration is 5 miles per hour per sec. for the increase in velocity in one second is 5 miles per hour.

a height, the acceleration is said to be *positive*; while, if the speed be decreasing, *e.g.*, when a railway train going at full speed approaches a halting station, or a stone is thrown upwards with a certain velocity, the acceleration is said to be *negative*. A negative acceleration is, in the ordinary language, called **Retardation.**

Acceleration may be uniform or variable. When the velocity of a body is increased by equal amounts in equal intervals of time, *however small* these intervals may be, the acceleration is said to be uniform. When this is not the case, the acceleration is variable.

Uniform acceleration is measured by the change in velocity that takes place in a given time divided by that time, *i.e.*, by the change of velocity in a unit of time. Hence, if the velocity of a particle changes from u to v during an interval of time t, the acceleration f is given by the equation, $f=(v-u)/t$ and this value is the same at all instants. When *variable*, the *acceleration at any instant* is also given by the equation $f=(v-u)/t$ with the only restriction that the interval of time t should, in this case, contain this particular instant and should be taken so small that the *acceleration* does not appreciably alter during this interval. It is obvious that the value of f is different at different instants.

28. Uniformly accelerated Rectilinear Motion.—We have already seen that the velocity of a body having an acceleration may change either in magnitude or in direction or in both. The simplest case, however, is that in which the magnitude of the velocity only changes at a constant rate and the motion is restricted in a straight line. This type of motion is called uniformly accelerated rectilinear motion. We shall now consider this type.

Equations of Motion.—Suppose that a body is moving with an initial velocity u and that it has a uni-

form acceleration f. After a time t reckoned from the start, the increase in velocity will be ft. If the final velocity at the end of the time t be v, then we have

v = initial velocity + increase of velocity due to the acceleration.

$$= u + ft \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

Since the acceleration, *i.e.*, the rate of increase of velocity is uniform, the average velocity v during the period t will be half-way between u and v and will be given by

$$v = \frac{u+v}{2} \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

This equation can be arrived at in the following way :

Divide the period of time t into a large number of intervals each of duration p, p being so small that the velocity at the beginning or the end of any interval may be taken to be the velocity throughout that interval.

Then we have :

	Velocity during successive intervals from the beginning of the period t onwards (velocity during any interval is taken to be the velocity at the beginning of that interval).	Velocity during successive intervals from the end of the period t backwards (velocity during any interval is taken to be the velocity at the end of that interval)
First,	u	v
Second,	$u+fp$	$v-fp$
Third,	$u+2fp$	$v-2fp$
...	...	...
*n*th.,	$u+(n-1)fp$	$v-(n-1)fp$
...	...	...
Last.	v	u

It will be seen that the velocity is different for different intervals. But the average velocity for any pair of intervals arranged in a line will be the same and will be equal to

$$\frac{u+v}{2}$$

For, we know that the average velocity during any period of time is measured by the total distance traversed divided by the total time. So that, the average velocity corresponding to the n th. pair of intervals is given by

$$\frac{\begin{array}{c}\text{the distance traversed}\\ \text{in the } n \text{ th. interval}\\ \text{from the beginning}\end{array} + \begin{array}{c}\text{the distance traversed}\\ \text{in the } n \text{ th. interval}\\ \text{from the end}\end{array}}{\text{total time comprising the two intervals}}$$

$$= \frac{\{u+(n-1)fp\}p + \{v-(n-1)fp\}p}{2p}$$

$$= \frac{u+v}{2}$$

In this way, it can be easily seen that the average velocity corresponding to any such pair of intervals is the same. Hence the average velocity v throughout the whole period of time t is given by

$$v = \frac{u+v}{2}.$$

The distance s travelled by the body in the time t will then be given by

$$s = \text{Average velocity} \times \text{time}$$

$$= \frac{u+v}{2} . t$$

and substituting the value of v from eqn. (4),

$$s = \frac{u+(u+ft)}{2} t$$

$$= ut + \tfrac{1}{2} ft^2 \qquad \dots \qquad \dots \qquad (6)$$

From eqn. (4),

$$v^2 = (u + ft)^2$$
$$= u^2 + 2uft + f^2t^2$$
$$= u^2 + 2f(ut + \tfrac{1}{2}ft^2)$$
$$= u^2 + 2fs \text{ from eqn. (6)} \quad \ldots \quad (7)$$

If the body starts from rest, we have $u = o$ and the general equations given above take the forms

$$v = ft \quad \ldots \quad \ldots \quad \ldots \quad (8)$$
$$s = \tfrac{1}{2}ft^2 \quad \ldots \quad \ldots \quad \ldots \quad (9)$$
$$v^2 = 2fs \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

If the motion is retarded instead of being accelerated, the general equations become

$$v = u - ft \quad \ldots \quad \ldots \quad (11)$$
$$s = ut - \tfrac{1}{2}ft^2 \quad \ldots \quad \ldots \quad (12)$$
$$v^2 = u^2 - 2fs \quad \ldots \quad \ldots \quad (13)$$

The distance passed over by a body in a particular second can be easily calculated by applying eqn. (6); we have

Distance traversed in the nth. second from start

= distance traversed in first n seconds − distance traversed in $(n-1)$ seconds.

$$= (un + \tfrac{1}{2}fn^2) - \{u(n-1) + \tfrac{1}{2}f(n-1)^2\}$$
$$= u + \tfrac{1}{2}f(2n-1) \quad \ldots \quad \ldots \quad (14)$$

If the body starts from rest, $u = o$ and distance traversed

$$= \tfrac{1}{2}f(2n-1) \quad \ldots \quad \ldots \quad (15)$$

EXAMPLES :—

1. A ball thrown along the surface of the ground with an initial velocity of 50 feet per sec. comes to rest after moving through 100 yds. Assuming the retardation constant find out how far the ball will have moved in 3 sec. and how long it had been in motion.

Let f be the uniform retardation and t the time for which the ball was in motion.

Then, since the ball comes to rest, the final velocity after t seconds from start is zero. Hence

$$0 = u - f.t$$
$$= 50 - f.t$$

Or $t = \frac{50}{f}$ seconds,

We also have

$$v^2 = u^2 - 2fs$$

or $0 = 50^2 - 2f \times 100 \times 3$

or $f = \frac{50^2}{2 \times 100 \times 3} f.t$ per sec. per sec.

$$\therefore t = \frac{50}{f} = \frac{50 \times 2 \times 100 \times 3}{50^2} \text{ seconds}$$

$$= 12 \text{ seconds}$$

Hence the ball was in motion for 12 seconds.

And the distance s travelled in 3 seconds is given by

$$s = ut - \tfrac{1}{2} ft^2$$

$$= (50 \times 3 - \tfrac{1}{2} \times \frac{50^2}{2 \times 100 \times 3} \times 3^2) \text{ ft.}$$

$$= 43\tfrac{3}{4} \text{ yds.}$$

2. A body, dropped from a height, falls with a uniform acceleration. In the 4th and 6th seconds from the commencement of its fall it moves through 120 ft. and 184 ft. respectively. Find its initial velocity and the acceleration with which it moves.

Let u be the initial velocity, and f the acceleration.

Then the distance traversed in the 4th. second = distance fallen through in 4 secs. − distance fallen through in 3 secs.

$$= [u.4 + \tfrac{1}{2}f\, 4^2] - [u.3 + \tfrac{1}{2}f.3^2]$$

$$= u + \tfrac{7}{2}f = 120 \quad \ldots \quad \ldots \quad (1)$$

Similarly, the distance fallen through in 6th sec.

$$= [u.6 + \tfrac{1}{2}f\, 6^2] - [u.5 + \tfrac{1}{2}f.5^2]$$

$$= u + \tfrac{11}{2} f = 184 \quad \ldots \quad \ldots \quad (2)$$

From (2) and (1)

$2f = 64$ whence $f = 32 ft.$ per Sec. per Sec.

and $u = 8$ $ft.$ per sec.

29. Graphical Representation of Motion.—The motion of a body can be conveniently studied by graphical means as follows. Let OX and OY (Fig. 19) be two axes at right angles to each other representing time and speed respectively. Let a number of equal divisions OA, AB, BC etc. along OX represent equal intervals of time, say one second. Suppose we

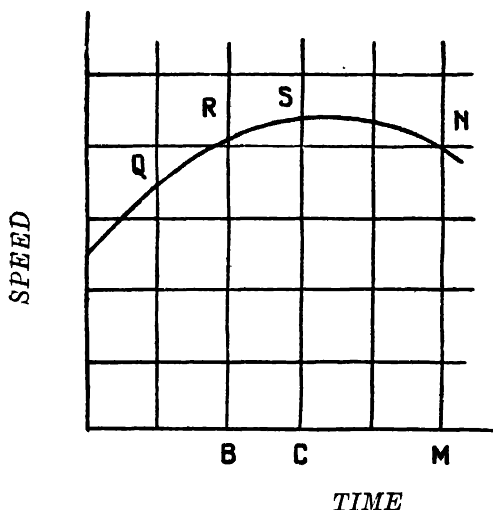


Fig. 19.
Speed-time diagram.

commence to count the time from O and let the speed with which the body is moving at this instant be represented by the length OP along the speed axis. Similarly, AQ represents the speed at the time OA, BR the speed at OB and so on.

If we assume that the speed of the body changes continuously from the beginning and not abruptly, then a smooth curve drawn through the points P, Q, R, S etc. will represent the actual motion of the body. For instance, the speed at any time represented by OM will be given by MN, the ordinate at this time. Such a curve is known as the speed-time diagram of the motion.

It must not be supposed, however, that the curve in such a diagram represents the actual path of the body. Motion might take place in a straight line, and yet the speed curve might be represented as in Fig. 19. The graph is merely a representation of the increase or decrease in the rate of motion at successive intervals of time.

It may be shown that the area under the speed curve, *i. e.*, the area bounded by the speed curve, the axis of time and the ordinates drawn through the points corresponding to the beginning and end of the interval considered gives the distance travelled during the interval. Thus in Fig. 19, the area $OPQRSCO$ is numerically equal to the distance traversed by the moving body in the time represented by OC. This will be clear from the following discussion.

Uniform Motion. – In case of uniform motion the speed at different periods of time remains the same. The speed time curve becomes a straight line parallel to the time-axis (Fig. 20).

Let the uniform speed of a body u be represented by the ordinate OP in the diagram. Suppose that the distance traversed by the body in the time t represented by OS is to be found out. As stated

before, it is numerically equal to the area OPRS.

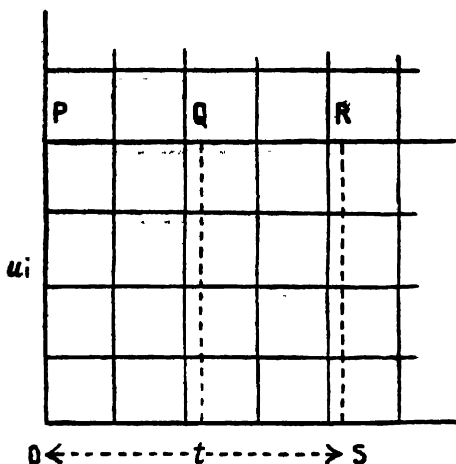


Fig. 20.
Speed-time diagram for uniform motion.

That is,

$$S = \text{area OPRS}$$
$$= OP \times OS$$
$$= u \times t$$

This is identical with eqn. (3).

Uniformly Accelerated Rectilinear Motion – In case of uniformly accelerated motion in a straight line, the speed increases by equal amounts in equal intervals of time and the speed-time curve is represented by a straight line inclined to the time axis.

Let v be the speed of a body at a certain instant of time represented by OA in the diagram in (Fig.21). Let f be the acceleration and v the velocity after a time t, represented by AD, has elapsed.

If AB, BC each represents one second and AP, BQ, CR represent the velocities at the instants A, B

and C, then since the acceleration is uniform, QL = RM = f, where PL, QM etc. are all parallel to AD. Hence, from geometry, P, Q and R are in a straight line. Proceeding in this way it may be shown that the speed-time curve of this motion is represented by this same line produced either way

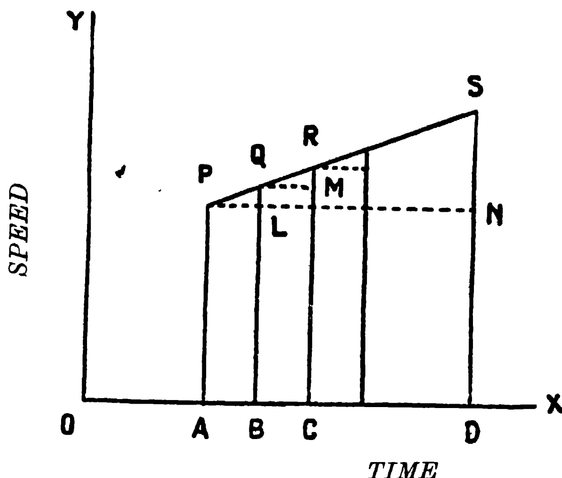


Fig. 21.
Speed-time diagram for uniformly accelerated rectilinear motion.

Let DS be the ordinate at D. Then we have DS = v

$$= DN + NS, \text{ where PN is parallel to AD}$$
$$= PA + NS$$
$$= u + ft. \text{ as in eqn. (4).}$$

for $\frac{NS}{NP} = \frac{QL}{PL}$, *i.e.*, $NS = NP. \frac{QL}{PL} = tf$

And the distance s traversed in time t is, given by

$$s = \text{area APSD} = \tfrac{1}{2}(AP + DS) \times AD$$
$$= \tfrac{1}{2}(u + u + tf) \times t = ut + \tfrac{1}{2}ft^2$$

This is identical with eqn. (6).

30. Composition of Velocities.—It has already been seen that velocity being a vector quantity can be represented by a straight line. For this purpose, a straight line may be taken in the direction of the velocity and of such a length that it contains as many units of length as there are untis of speed in the velocity considered. In order to show the *sense* of the direction, an arrow-head may be drawn on the line.

A body may have at the same time more than one velocity. For example, a man moving about in a train has not anly the velocity of the train, but an additional velocity of his own. Again, a person walking on the deck of a ship has an actual velocity which may be said to be compounded of three velocities *viz.*, that of the ship, of the current and of his own on the deck. When a body has two or more simultaneous velocities, the velocity with which it actually moves is called the RESULTANT velocity, and those several velocities are called the COMPONENTS. The process of finding the resultant velocity when the component velocities are given, is called *compounding* the velocities.

To compound velocities in the same straight line—

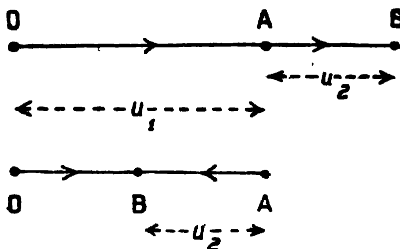


Fig. 22.
Velocities in the same line.

If a body moves with several uniform velocities in

the same straight line, the resultant velocity will be the algebraical sum of the component velocities. Thus, if a body tends to move from O to A in one second (Fig. 22) due to a velocity u_1 and from A to B in the same line in one second due to a second velocity u_2, then at the end of one second the body will be found at B, as if it had moved with a velocity $u_1 \pm u_2$. Suppose the velocity of a stream is 2 miles per hour and a man can row a boat through still water at 8 miles an hour, then the actual velocity of the vessel is 6 or 10 miles an hour according as the vessel is sailing up or down the stream.

In the case of variable velocities, *i.e.*, velocities with acceleration, in the same line, the above remark is also true.

To compound velocities, not in the same straight line – If a body has two different velocities in different directions at the same time, it is evident that the actual motion of the body is along neither of these directions, but is along a line intermediate between them. Thus, if a man rows a boat at right angles to the current of a river, the actual course of the boat is a line crossing the river in an oblique direction from one bank to the other.

Let the two simultaneous velocities be represented by the lines OA and OB (Fig. 23), and let their magnitudes be u and v. Now we may imagine the body moving with the velocity u along OA, while the line OB moves with velocity v parallel to itself. At the end of one second, the particle would be at A due to

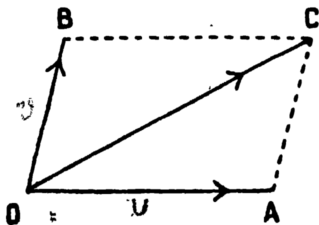


Fig. 23.
Parallelogram of Velocities

the first motion, but owing to the motion of the line

OA, the point A at the end of one second will come to C. Thus the body will be at C.

Now, since the component velocities are uniform *i.e.*, constant in magnitude and direction, their resultant must be a uniform velocity. In other words, the velocity of the body from O to C must also be constant in magnitude and direction. Hence the straight line OC represents the resultant velocity.

We have, from the above, a rule for finding the resultant of two volocities, known as the **Parallelogram Law.** *If a particle possesses simultaneously two velocities represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, they are equivalent to a single resultant velocity, also represented in magnitude and direction, by the diagonal of the parallelogram passing through that point.*

Expression for the Resultant Velocity.—Let us find out expressions for the magnitude and direction of the resultant of two velocities u and v inclined at an angle α (Fig. 24). From the law of parallelogram of velocities OC represents the resultant. Draw CD perpendicular to OA produced.

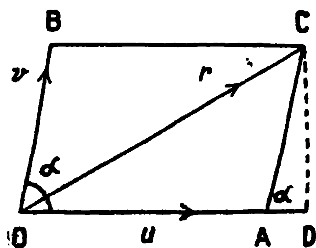


Fig. 24.
Composition of Velocities

Then

$\angle BOA = \angle CAD = \alpha$

Now

$$OC^2 = OD^2 + CD^2$$
$$= (OA + AD)^2 + CD^2$$
$$= OA^2 + 2\,OA.AD + AD^2 + CD^2$$
$$= OA^2 + 2OA.AC \,.\, \cos\alpha + AC^2 \qquad \dots (16)$$

If the resultant OC be denoted by r, we have

$$r^2 = u^2 + v^2 + 2uv \operatorname{Cos} \alpha \quad \ldots \quad \ldots \quad (17)$$

$$\text{Also } \tan \text{COA} = \frac{\text{CD}}{\text{OD}} = \frac{\text{CA } \sin \text{CAD}}{\text{OA} + \text{CA } \cos. \text{CAD}}$$

$$= \frac{v \operatorname{Sin} \alpha}{u + v \operatorname{Cos} \alpha} \quad \ldots \quad (18)$$

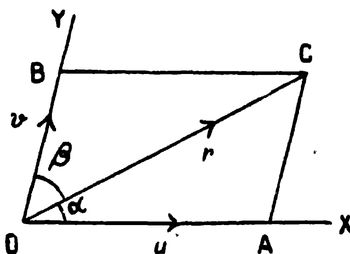
Thus we see that the resultant of two velocities u and v inclined at angle α is given by $\sqrt{u^2 + v^2 + 2uv \operatorname{Cos} \alpha}$ and is inclined at an angle $\tan^{-1} \frac{v \operatorname{Sin}\alpha}{u + v \operatorname{Cos}\alpha}$ to the direction of the velocity u.

If a body possesses simultaneously more than two velocities, the resultant velocity may be obtained by the repeated application of the parallelogram law. At first, the resultant R of any two of velocities is found ; then the resultant R_2 of R and another of the given velocities, and so on. The last resultant so found is the resultant of all the velocities considered.

31. Resolution of a Velocity.—It is sometimes convenient to replace the actual velocity of a body by two or more other velocities. In such a case, the velocity is said to be *resolved* into component velocities. Let a velocity r be represented by a straight line OC (Fig. 25 a). It is required to find its components, one along a direction OX and the other along any other direction OY. Through C draw CA parallel to OY to meet OX in A and CB parallel to OX to meet OY in B. Then, since OBCA is a parallelogram, the two velocities u and v, represented by OA and OB respectively, would have a resultant OC, so that we may replace the velocity r by the two velocities u and v. Any given velocity may be resolved in an infinite number of ways.

If α and β be the inclinations to the resultant r of

the components u and v respectively, then we have, from geometry,



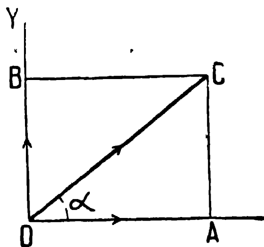


Fig. 25(a) Fig. 25(b)
Resolution of Velocities.

$\angle$ OCA $=\beta$; and in the triangle OAC,

$$\frac{\text{OA}}{\text{Sin OCA}} = \frac{\text{AC}}{\text{Sin AOC}} = \frac{\text{OC}}{\text{Sin OAC}}$$

$$i.e., \quad \frac{u}{\text{Sin}\,\beta} = \frac{v}{\text{Sin}\,\alpha} = \frac{r}{\text{Sin}\,\{\pi - (\alpha+\beta)\}}$$

$$= \frac{\quad}{\text{Sin}\,(\alpha+\beta)} \qquad \ldots \quad (19)$$

These equations give the magnitudes of the components of a given velocity when their directions are given.

The most important case in practice, however, is that in which the two components are at right angles to each other as in fig. 25 (b) and then each component is said to be the *resolved part* of the force in the corresponding direction.

Then $\beta = \frac{\pi}{2} - \alpha$ and $\alpha + \beta = \frac{\pi}{2}$; so that, from eqn. (19), $u = r$ Cos α and $v = r$ Sin α, for Sin $(\alpha+\beta) = 1$ and Sin $\beta =$ Cos α

32. Relative Velocity.—It is sometimes desirable to determine the motion of a body relative to a second body which is itself in motion.

The relative velocity of one particle A with respect to a second particle B is the rate *at which A changes its position relative to B.*

If the two particles are in motion with the same velocity, there is no change in their relative position. Hence the velocity of one, relative to the other, is zero. Thus a person in a train would, if he kept his attention fixed on a passenger in another train moving in the same direction with same velocity, and if he were unconscious of his own motion, considers the second person to be at rest.

When the particles are in motion with different velocities, the relative velocity of A with respect to B is obtained by compounding with the velocity of A, a velocity which is equal and opposite to that of B.

Let the velocity of the particle A in Fig. 26 be represented in magnitude and direction by AP and that of B by AQ. Now it is clear that relative motion of two particles is not altered, if we impress the same velocity on both the particles ; for example, the relative motion of two flies crawling on the window of a railway train is the same, whether the carriage be at rest or in motion.

Imagine, therefore, that a velocity equal and opposite to that of B is impressed on the motions of A and B, represented by AR for the particle A and BR for the particle B. The relative motion is unaltered. The particle B is brought to rest, while A moves with a velo-

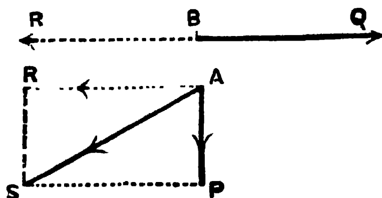


Fig. 26.
Relative Velocity.

city which is the resultant of its own velocity and the reversed velocity of B. The apparent velocity of A is represented in the fiig. 26 by AS.

When a man is running in a horizontal direction amidst a shower of rain, the drops which are falling, say vertically, appear to him to strike his face in a slanting direction. The explanation of this is clear, if we apply to each a horizontal velocity equal and opposite to that of the man. This brings the man to rest. The resultant velocity of a rain-drop is its apparent or relative velocity, in magnitude and direction, with respect to the man.

33. Angular Velocity.—When a rigid body rotates round any straight line as its axis of rotation, all the particles, of which it may be regarded to be built up, move in circles around points on the axis as centres.

The **angular Velocity** of rotation is measured by the angle which the radius, joining any particle to the centre of its path of motion, describes per unit of time. This angle is evidently the same for every particle in the body.

The angular velocity is said to be *uniform* when equal angles are swept through in equal intervals of time; otherwise it is *variable*.

Let a body moving with uniform angular velocity ω describe an angle θ in t seconds. Then

$$\omega = \frac{\theta}{t} \text{ and } \theta = \omega t \qquad \dots \qquad \dots \quad (20)$$

The angle θ is always expressed in circular measure, *i. e.*, in radians. If s be the arc described in time t by a particle situated at a distance r from the axis, with a uniform speed v, then

$$\theta = \frac{s}{r} = \frac{vt}{r}$$

$$= \omega t \text{ from eqn. } (20)$$

$$\text{whence } v = r\omega \quad \dots \qquad \dots \qquad \dots \quad (21)$$

The linear velocity of any particle in the body is thus directly proportional to its distance from the axis of rotation, and is found by multiplying the angular velocity by this distance. Thus in the case of a rotating wheel, a point on the rim has a greater speed than one on a spoke. It is also evident from the fact that the former being situated at a greater distance from the axle has to describe a larger circle in exactly the same time in which the latter finishes its revolution.

The angular velocity may be expressed in terms of the number of turns per second. Let the body make n turns per second. Since in one complete turn the angle turned through is 2π, the angle turned through in one second or the angular velocity is given by

$$\omega = 2\pi n \qquad \ldots (22)$$

If T is the time taken for one complete revolution, we see that

$$\omega T = 2\pi$$

$$Or\ \omega = \frac{2\pi}{T} \qquad \ldots (23)$$

34. Motion in a Curved Path.—If a particle moves along a curved path, its direction of motion, which, at any instant is along the tangent to its path, changes from point to point. Hence its velocity alters continuously, even if the magnitude of the velocity remains constant.

Consider a particle moving along a curved path ABCD (Fig. 27). Let the velocity v_1 of the particle as it passes the point A be represented by the straight line AP in a direction tangential to its path at A. At the point B, the velocity has a different value v_2 represented by BQ in direction and magnitude. Similarly let v_3 and v_4 represent the velocities at the points C and D.

From a point O draw straight lines Oa, Ob, Oc,

Od to represent the velocities v_1, v_2, v_3, and v_4 both in magnitude and direction. Join ab, bc, cd.

The straight lines ab, bc, cd represent the change of velocity occurring during the intervals between A

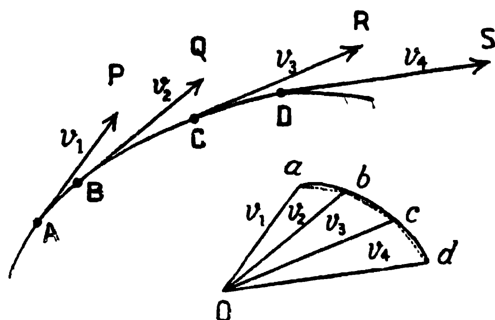


Fig. 27
Motion in a curved path . hodograph.

and B, B and C, C and D respectively. If t_1, t_2, t_3 represent the times taken by the particle to move from A to B, from B to C and from C to D, then the average acceleration during the respective intervals are,

$$\frac{ab}{t_1}, \frac{bc}{t_2}, \text{ and } \frac{cd}{t_3}.$$

If the points A, B, C and D are taken close together, the intervals t_1, t_2, t_3 become very small and $\frac{ab}{t_1}$, $\frac{bc}{t_2}$, and $\frac{cd}{t_3}$ then represent instantaneous accelerations of the particle at these points. A smooth curve drawn through a, b, c, and d would then coincide with the lines ab, bc, etc. Such a curve is called the **hodograph** of the motion of the particle.

The acceleration of a body at any instant may

therefore be obtained from the hodograph of its motion by taking a short length of the hodograph at the instant under consideration and dividing it by the interval corresponding to this length.

Uniform motion in a Circle.—When a particle is moving *uniformly* in a circle, there is no tangential acceleration for the magnitude of the velocity is constant. The only acceleration which it possesses, is always at right angles to the direction of motion, *i.e.*, directed towards the centre of the circle. This **normal acceleration** causes the direction of the velocity of the particle to change continuously from point to point on the circle.

The magnitude of the normal acceleration can be found out as follows :—

Let a particle move with uniform speed v in a circular path (Fig. 28) of radius r. Suppose that at a certain instant the particle is at A and that it moves from A to B in a short interval of time t. It has a

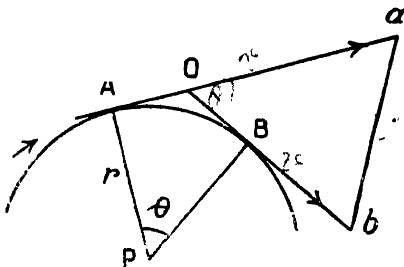


Fig. 28.
Uniform motion in a circle.

velocity v along the tangent at A. Let the straight line Oa represent this velocity. At B also the velocity of the particle is v but now directed along the tangent at B. Let Ob represent this velocity. Then ab represents the change in the velocity in time t.

Hence the average acceleration f between A and B is given by

$$f = \frac{a\,b}{t} \quad \ldots \quad (24)$$

Now the angle θ between the radii PA and PB is equal to the angle between the tangents to the circle at A and B, that is

$$\theta = \angle aOb.$$

Now since t is very small B is very near to A and $\angle aOb$ is very small. In that case ab becomes equal to the arc of a circle described about O as centre and v as radius. Hence,

angle aOb in circular measure $= \frac{a\,b}{v}$

Also θ in circular measure $= \frac{\text{AB}}{r}$ (*cf.* §21)

$$\therefore \quad \frac{\text{AB}}{r} = \frac{a\,b}{v}$$

$$\text{or} \quad \frac{v\,t}{r} = \frac{f\,t}{v} \text{ from eqn.} \quad (24)$$

$$\text{or} \quad f = \frac{v^2}{r} \quad \ldots \quad \ldots \quad (25)$$

Here f is the average acceleration between A and B. But when, as has been mentioned above, t is indefinitely small, f becomes the instantaneous acceleration of the particle at A, *i.e.*, at any point on the circle. Since it is always normal to the path of the particle, it is always directed towards the centre of the circle.

35. Simple Harmonic Motion.—Let a point P (Fig. 29) move in a circle of radius r and centre C with uniform speed v. Let a perpendicular PN be dropped from it on any diameter AB of the circle. Then, as the point P travels round the circle, the point N moves to and fro along AB. The motion of N is then called Simple Harmonic Motion or as it is usually abbreviated, S. H. M. It is, therefore, defined as the projection of a uniform circular motion upon a straight line.

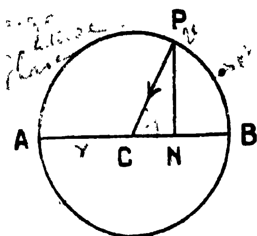


Fig. 29
Simple Harmonic Motion

Examples of such motion are those of the bob of a pendulum, the prongs of a vibrating tuning fork and so on.

It is evident that the time taken by P to complete one revolution is the same as the time taken by N to go from one end of the diameter AB to the other end and back. This time is called the **period** of the S. H. M. Each complete to and fro movement is called an oscillation and the number of such oscillations made in one second is called the **frequency** of the S. H. M. The whole distance which N covers between the two extreme positions, *i.e.*, AB, is called the **range** of the S. H. M. and half of the range *i.e.*, CB is called the **amplitude**. The angle PCN is called the **phase** at the instant considered. We see, therefore, that during one complete period the phase varies from 0 to 2π.

At any particular instant, the velocity of N is the resolved part of the velocity of P parallel to the diameter AB. Similarly the accelaration of N at any instant is the component of the accleration of P parallel to AB.

The acceleration of P is equal to $\frac{v^2}{r}$ and is always directed towards the centre C (cf. § 34). Let this acceleration be represented both in magnitude and direction by the radius PC. Then its component parallel to AB will be represented by NC and its magnitude will be given by

$$\text{Acceleration of N} = \frac{v^2}{r} \times \text{Cos PCN}$$

$$= \frac{v^2}{r} \times \frac{NC}{PC}, \quad \text{(from the Parallelogram Law.)}$$

The distance of N from the central or mean position C is called the **displacement** of the point N. Thus we have

$$\text{Acceleration of N} = \frac{v^2}{r} \times \frac{\text{displacement}}{r}$$

$$= \frac{v^2}{r^2} \times \text{displacement}$$

But since $v = r\omega$ and $\omega = \frac{2\pi}{T}$ (cf. — § 33), we get

$$\frac{\text{Acceleration}}{\text{Displacement}} = \omega^2 = \frac{4\pi^2}{T^2} \qquad \ldots (26)$$

$$= \text{Constant.}$$

Hence we arrive at the inportant theorem :

The acceleration of a particle undergoing simple harmonic motion is always directed towards the mean position and is proportional to the displacement of the particle from that position.

Exercise II

1. Define motion. Name and explain the different forms of motion.

2 Distinguish between Speed and Velocity. What is Angular Velocity ? [C.U—1930]

3. Define Uniform Velocity and explain how it is measured.

4. What is meant by the statement that the acceleration of a particle is 32 ft per sec. per sec. ?

5 A river, 1 mile broad, is running downwards at the rate of 4 miles an hour, and a steamer, moving at the rate of 8 miles an hour, wishes to go straight across. How long will the steamer take to perform the journey, and in what direction must she be steered ? [*Lond Matric*]

6. A train, moving with a velocity of 30 miles an hour, stops in 5 minutes as it approaches a station. Express the retardation assuming it to be uniform, and taking a foot and a second as the units of length and time ?

7. A ball is thrown out of a window of a railway carriage in motion. In what direction will it seem to fall to a passenger in the train and also to a person on the ground ?

8 At the earth's equator the hot air ascends and is replaced by cold air which blows in along the ground from the poles. That which comes from our hemisphere blows from the north-east instead of from the North Explain this [*Lond Matric*]

9. A man is walking in the north-easterly direction with a velocity of 6 miles per hour Find the components of his velocity in directions due north and due east respectively

10. Show how to find by graphical construction drawn to scale and also by calculation the resultant of the following velocities communicated to a point, viz., 2 ft per second in an easterly, 20 ft. per second in a north-easterly, and 30 ft per second in a northerly direction respectively

11 Define the angular velocity of a body moving uniformly in a circle. Find its periodic time

Show that the foot of the perpendicular drawn from the body to a fixed diameter of the circle describes a S. H M. and hence define such a motion. [*C. U.—1933.*]

12. Explain what is meant by a hodograph. Describe the hodograph of the motion of a particle moving uniformly in circle.

13 Explain S. H. M. and give two examples illustrating your answer. Explain the terms, period, phase and amplitude. [*C U—1935*]

PART IV

FORCE

CHAPTER—IV

FORCE.

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36. Force—A material body is said to be inert, *i.e.*, it cannot *of itself* move when it is at rest, nor can it come to rest when it is already in motion. To change its state of rest or of uniform motion in a straight line it must be acted on by some external agency. **Force** is the name given to this agency and may be defined as whatever causes or tends to cause motion or change of motion in a body.

When one kicks a football he exerts muscular force on it which makes it move. When a ball is running towards the goal somebody has to exert force on it to change its course. The direction and magnitude of the velocity with which the ball subsequently moves depends on the direction and magnitude of the force and also on the particulur point at which it is applied. This latter is called the point of application of the force.

Representation of a Force—In order to define a force completely, it is necessary to specify

(i) its **Point of Application,**
(ii) its **Direction** and
(iii) its **Magnitude.**

It is, therefore, a vector quantity and, like a velocity, can be represented by a straight line, for

(i) one end of the line *may* represent the point of application of the force ;

(ii) the direction of the line gives the direction or line of action of the force. The *sense* of the

direction may be indicated by an arrow-head drawn on the line ; and

(iii) the number of unit of length on the line gives a measure of the magnitude of the force.

37. Composition of Forces acting at a point :—If two or more forces act at a point in a body, they can always be compounded into a single resultant which is equivalent to all component forces. The method of composition of forces is exactly similar to that of velocities.

Forces Acting in a Straight Line.—If two forces, say P and Q, act at a point *along the same straight line*, their resultant R is obviously equal to their sum, if they act in the same direction, or equal to their difference, if they act in opposite directions ; in the latter case, the resultant acts in the same direction as that of the greater force.

If a number of forces act at a point along the same straight line, the forces acting in one direction may be considered positive and those in the opposite direction negative, and then the resultant of the forces is given by their algebraic sum.

Forces inclined to one another.— If two forces, say P and Q act at a point in directions inclined to each other, their resultant is at once obtained by *the* **parallelogram law**.

The law of parallelogram of forces states that :

If two forces acting at a point be represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is represented both in magnitude and direction by the diagonal of the parallelogram passing through that angular point.

Thus let any two forces P and Q acting at a point O be represented both in magnitude and direction by

the two straight lines OA and OB. Complete the parallelogram OA-CB with OA and OB as the adjacent sides. Their resultant R is represented both in magnitude and direction by the diagonal OC passing through O, [Fig. 30].

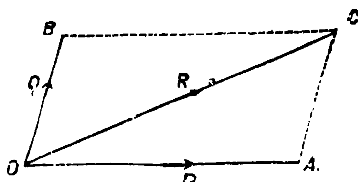


Fig. 30
Parallelogram of forces

The Magnitude of R :—

The *magnitude* of the resultant may be obtained graphically by measuring the diagonal OC on the same scale on which OA and OB are measured. We also have, from Geometry,

$$OC^2 = OA^2 + OB^2 + 2OA.\ OB \cos AOB$$

Substituting, we have

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha \quad \ldots \quad \ldots \quad (27)$$

where α is the angle between the directions of the two forces.

If in a particular case the forces act at right angles to each other, we have

$\alpha = 90^\circ$ and $\cos 90^\circ = 0$.

Hence, the expression for R in the formula reduces to

$$R^2 = P^2 + Q^2. \quad \ldots \quad \ldots \quad (28)$$

These equations give the magnitude of the resultant when the component forces and their inclinations are known.

When more than two forces act at a point the resultant can be calculated first by compounding any two of the forces in the above way and finding their resultant R_1. This latter is then compounded with another of the given forces to yield a second resultant R_2 and so on. The final resultant R, when all the

component forces are exhausted, gives the resultant of the whole system of forces.

Practical Verification :

The truth of the law of parallelogram of forces may be verified experimentally in the following manner :

Expt. I Take three strings and knot them together at a point O Attach to their ends any three weights P, Q and R (any two of which are together greater than the third). Let one string be suspended freely with its weight R, and let the other two pass over two smooth light pulleys P_1 and P_2 placed at the corner tops of a vertical board as in fig. 31.

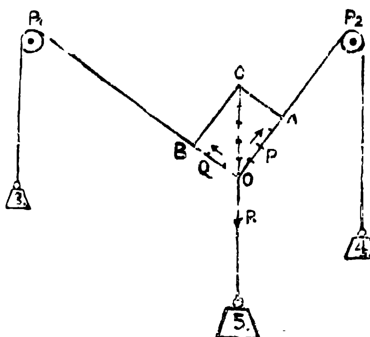


Fig. 31
Illustrating the Parallelogram of forces

When the suspended system assumes a position of equilibrium, mark on a piece of paper fixed on the board, the directions of P, Q and R. Take off lengths OA and OB proportional to the magnitudes of P and Q respectively. Draw OC in a direction opposite to that of R so that the length OC is proportional to the magnitude of R on the same scale, *i.e.*, OC represents a force equal in magnitude but opposite in direction to the weight R.

Now since the point O is in equilibrium under the action of the three forces P, Q and R, the resultant of P and Q must be equal and opposite to R. In other words, the resultant of the forces represented by OA and OB must be represented by OC.

Join CA and CB. It will be found that the figure OACB is a parallelogram of which OC is the diagonal passing through O. Hence the law is proved.

38. Triangle of Forces.—When two or more forces acting at a point are in equilibrium, *i.e.*, do not produce motion, then any one of these forces is said

to be the equilibrant of the remaining ones. Evidently the equilibrant of a number of forces must be equal and opposite to their resultant.

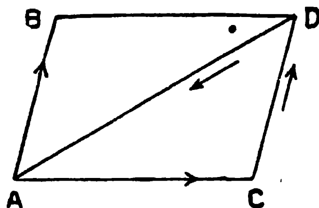


Fig. 32
Triangle of forces

Let AB and AC (Fig.32) represent two forces. Complete the parallelogram ABDC ; join AD. From the parallelogram law the diagonal AD represents the resultant of these forces AB and AC. Hence DA represents their equilibrant. That is, the forces AB, AC and DA are in equilibrium.

Now ABDC being a parallelogram, CD is equal and parallel to AB. Hence the force represented by AB may also be represented by CD both in magnitude and direction but not in position. Consequently the three forces represented by AC, CD and DA are in equilibrium. We thus arrive at the important theorem.

If three forces acting on a particle can be represented both in magnitude and direction by the three sides of a triangle taken in order, then the forces are in equilibrium.

This proposition is known as the triangle of forces.

39. Polygon of Forces.—Let AB, BC, CD and DE in fig. 33 represent in magnitude and direction four forces acting at a point. Join AC. Then by the triangle of forces CA represents the equilibrant of AB and BC. In other words AC is the resultant of AB and BC. Similarly AD is the resultant of AC and CD, *i.e.*, of AB, BC and CD. In the same way AE is the resultant of AD and DE *i.e.*, of AD, BC, CD and DE. Hence the force represented by

EA must be the equilibrant of the four forces AB, BC, CD and DE.

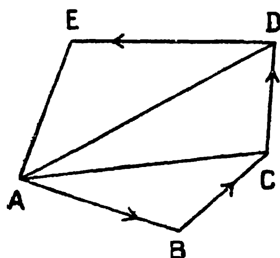
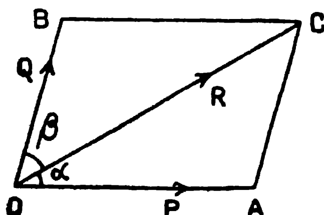


Fig. 33
Polygon of forces

We thus arrive at the law known as the polygon of forces, which is stated as follows.

If any number of forces acting on a particle can be represented in magnitude and direction by the sides of a closed polygon taken in order, then the forces will be in equilibrium. If the lines representing the forces do not form a complete polygon, then the forces will have a resultant which will be represented by the line joining the open ends, the direction being from the open end of the first side towards that of the last.

40. Resolution of Forces.—Just as we can compound two or more forces into a single resultant, so conversely we can resolve a single force into a number of *components*, which are together equivalent to it.



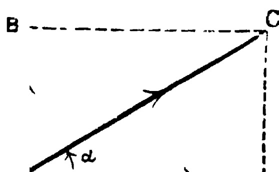


Fig 34(a) Fig. 34(b)
Resolution of forces

To resolve a force into its components in any two given directions, draw a straight line OC to represent the given force R (Fig. 34 a). Through O draw OA

parallel to one of these directions and through C, draw CA parallel to the other. Now complete the parallelogram : then, by the parallelogram rule, the forces OA and OB have OC for their resultant and are, therefore, the required components.

Let the component forces represented by OA and OB be denoted by P and Q respectively. If α and β be the inclinations of OA and OB to OC, then, from geometry,

$$\frac{OA}{\text{Sin } OCA} = \frac{AC}{\text{Sin } COA} = \frac{OC}{\text{Sin } OAC}$$

But $AC = OB$

$\angle COA = \alpha$

$\angle OCA = \beta$

and $\angle OAC = \pi - (\alpha + \beta)$

$$\therefore \frac{OA}{\text{Sin } \beta} = \frac{OB}{\text{Sin } \alpha} = \frac{OC}{\text{Sin } [\pi - (\alpha + \beta)]}$$

$$= \frac{OC}{\text{Sin } (\alpha + \beta)}$$

i.e., $$\frac{P}{\text{Sin } \beta} = \frac{Q}{\text{Sin } \alpha} = \frac{R}{\text{Sin}(\alpha + \beta)} \quad \ldots \quad (29)$$

These equations give the magnitude of the components when that of the given force and the respective inclinations of the required components to it are given.

The simplest and the most important case of the resolution of a force is that in which the two components are at right angles to each other. In that case (Fig. 34 b),

$\alpha + \beta = \pi/2$

and $\sin \beta = \sin (\pi/2 - \alpha)$

$= \text{Cos } \alpha$

$$\therefore \frac{P}{\text{Cos } \alpha} = \frac{Q}{\sin \alpha} = R$$

i.e., $P = R \text{ Cos } \alpha$ and $Q = R \sin \alpha$. ... ... (30)

EXAMPLES :

As an example of the resolution of forces, we may take the case of a boat sailing partly against the wind. Let AB (Fig. 35) be the direction of the boat's length, CD the direction of the sail and WC that of the wind. We take CE to represent the force the wind would exert on the sail, if it were placed at right angles to the direction of the wind. Resolve this into two components, one perpendicular to CD, and the other along CD. The former component CF only is efficacious in exerting a pressure on the sail ; the latter being directed along the surface of the sail has no effect on it. Now resolve the force CF along and across the boat. The component CK along the boat causes it to move forward, while the component CH across the boat tends to make it move through water in a direction perpendicular to its length.

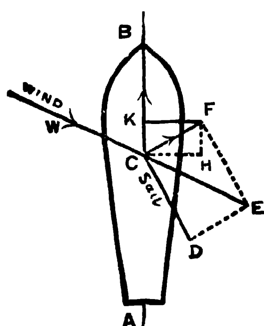


Fig. 35.

The sailing of a boat.

A kite at rest in the air is another example. The kite shown

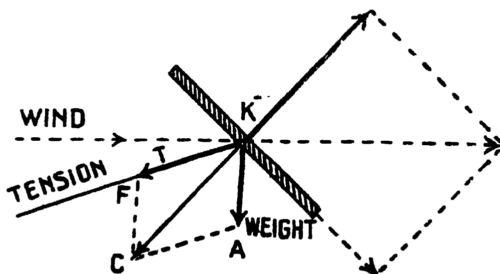


Fig. 36.

Forces acting on a kite at rest in the air.

in section (shaded) in fig. 36 is acted on by two downward forces, one is due to its own weight represented by KA and

the other due to the pull T along the string represented by KF. The resultant of these two forces is represented by KC acting downwards. The force of the wind represented by DK on the kite may be resolved into two forces, along and perpendicular to the face of the kite. The former component being along the face of the kite exerts no force on it. It is only the latter component, *i.e.*, the one perpendicular to its face which tends to move the kite up. If this equals the resultant KC, the kite is at rest ; if it be greater, the kite rises ; if less, the kite falls (Fig. 36).

41. Moment of a Force :—If a body be so situated that a line in it is kept fixed, so that the body cannot move out of its own place, *i.e.*, cannot undergo *translation*, but is free to rotate about that line, then any force applied to it in any direction not passing through the line will cause *rotation*.

The rotatory effect of a force depends on

(i) the magnitude of the force,

and (ii) the perpendicular distance of its line of action from the fixed axis, generally called the **axis of moment**.

Consider, for example, a door turning about its hinges. It requires a much smaller force to open or shut the door, if the force be applied in a direction perpendicular to the door at a considerable distance from the hinge outwards than if applied near to the hinge.

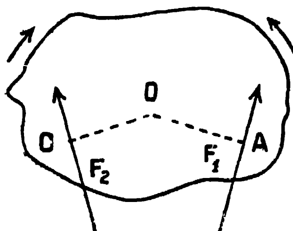


Fig. 37.
Moment of force.

The tendency of a force to produce rotation about a fixed axis is called its **moment** and is measured by the product of the magnitude of the force and the

length of the perpendicular from the axis to the line of action of the force. This latter is called the **arm** of the force. Thus if F_1 be the force acting on a rigid body capable of rotating about an axis through O perpendicular to the paper and OA be drawn perpendicular to the line of action of F_1, then the moment of F_1 = Force × arm = $F_1 \times$ OA.

Hence the moment of a force about a point vanishes when either

(i) the force itself is zero, *i.e.*, $F_1 = 0$

or (ii) the line of action of the force passes through the point, *i.e.*, the arm = 0.

Further, the moment of a force is regarded as *positive*, if it tends to produce rotation in the anti-clockwise direction, and as *negative* if it tends to produce rotation in the same direction as the hands of a watch move. Thus in fig. 37 the moment of F_1 about O is *positive*, while that of F_2 about O is *negative*.

Principle of Moments – It is evident from the above that if a rigid body, movable about a fixed point, is kept in equilibrium by two forces in any plane containing that point, the moments of these forces about the point will be equal and opposite.

In general, if a body be at rest under the action of several forces in the same plane, the total moment of all the forces about every point in the plain is zero, *i.e.*, the clockwise moments are equal to the anti-clockwise moments. This principle is known as the **law of moments** and is of great use in finding the magnitude or direction of some unknown force acting on a body which is at rest.

42. Composition of Parallel Forces—Forces whose lines of action are parallel, are called **parallel forces**. They are of frequent occurrence in practical mechanics.

Parallel forces acting in the same direction are

said to be **like** and those acting in opposite directions are said to be **unlike.**

The resultant R of two *like* parallel forces P, Q, acting at A and B respectively, must evidently be equal to their sum, *i.e.*, R = P + Q, and must act in the same direction, the line of action of the resultant lying between those of the component forces. It can be proved that its line of action meets the line AB in a point G, such that

$$P \times AG = Q \times BG$$

$$\text{Or} \quad \frac{P}{Q} = \frac{BG}{AG}$$

In other words, the point G divides the line AB *internally* (Fig. 38a) in the inverse ratio of the forces.

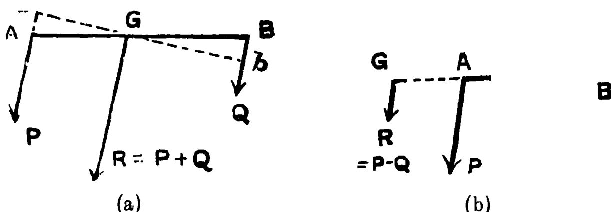


Fig. 38
Composition of parallel forces

The resultant R of two *unlike* parallel forces P, Q, acting at A and B respectively, is equal to their difference *i.e.*, R = P − Q (assuming P to be greater than Q), and acts in the direction of the greater force P. It may be proved that it acts at a point which divides AB *externally* (Fig. 38b) into parts which are in the inverse ratio of the forces, *i.e.*,

$$P \times AG = Q \times BG$$

It follows that the resultant is, in both the cases, nearer to the greater force.

In both the cases, draw a line aGB through G in a direction perpendicular to the line of action of the parallel forces, so that a and b are on the lines of action of P and Q respectively. Then, from similar triangles, AGa and BGb, we have

$$\frac{AG}{BG} = \frac{aG}{bG}$$

or
$$\frac{P}{Q} = \frac{bG}{aG}$$

or
$$P \times aG = Q \times bG \qquad \ldots \qquad (31)$$

i.e., the moments of P *and* Q *about* G *are equal and opposite.*

EXAMPLE :—

Two masses weighing 3 and 7 gms. are suspended from a uniform weightless rod 1 metre long. At what point will it balance ?

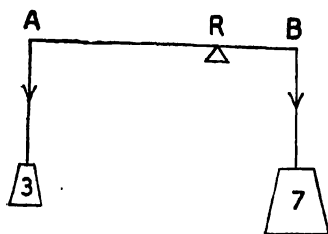


Fig. 39

Balancing a rod

Let AB (Fig. 39) be the rod, the weights 3 and 7 gms. being suspended from the extremities A and B respectively. Let R be the point at which it will balance. Then, obviously, the resultant of the two weights must pass through R. Hence,

$3 \times AR = 7 \times BR$
$= 7 \times (100 - AR)$,

for $AB = 100$ cms.

whence

$AR = 70$ cms.

Thus the rod will balance at a point 70 cms. distant from the extremity from which the weight of 3 gms. is suspended.

43. Couple—In the previous article if the two unlike parallel forces be equal in magnitude, the resultant of P and Q is zero. So no translatory motion is possible. Such a pair of unlike equal parallel forces

constitute what is called a **couple.** The couple may have, however, a moment about any point so as to produce rotation.

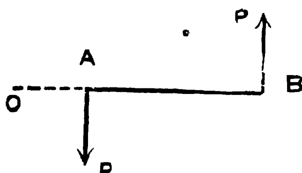


Fig. 40

Let O (Fig. 40) be a point either between P and P, or outside them. Draw OAB perpendicular to the forces. Then the algebraic sum of the moments of the two forces P and P about O

$$= P.\ OB \pm P.OA$$
$$= P.\ (OB \pm OA) = P.AB \quad \ldots \quad (32)$$

the perpendicular distance, AB, between the forces is called the **arm** of the couple and is a constant.

Hence the **moment of the couple** about O is independent of the position of O, and vanishes only if either a force of the couple or the arm vanishes.

As an example of couple action, we may cite the case of the earth's magnetic action on a magnetic needle. If the needle be displaced from the magnetic meridian, it quickly returns to it indicating thereby that the earth's action on it is in the nature of a couple.

Exercise III

1. The greatest resultant of two forces is 31 lbs. and the least is 17 lbs ; what is the resultant when the two forces act at right angles to one another ?

2. Forces 6 lbs., 8 lbs., and 10 lbs. act at a point in directions parallel to the sides of an equilateral triangle taken in order. Find their resultant.

3. A man carries a bundle at the end of a stick over his shoulder. If the distance between his hand and the bundle be kept constant, and the distance between his hand and shoulder be varied, how does the force on his shoulder change ?

4. Two men carry a load of 1 cwt. suspended from a horizontal uniform pole 12 ft long, whose weight is 20 lbs. and whose ends rest on their shoulders. Where must the load be suspended in order that one of them may bear 94 lbs. of the whole weight?

5. A uniform rod, 4 ft. long and weighing 16 lbs., rests on a horizontal table with one end projecting 8 inches over the edge. Find the greatest weight which can be hung on the end without making the rod topple over.

6. Explain how a kite rests in air.

7. Define moment of a force about a point. What is the principle of moments?

8. What is a couple? Show that the moment of a couple is independent of the position of the point about which the moment is considered.

PART V

THE LAWS OF MOTION

CHAPTER V

THE LAWS OF MOTION

So far we have dealt with the motion of bodies and with the forces acting on them quite apart from one another. In Chapter III we considered motion without any reference to the force producing it and in the last Chapter we considered force quite apart from the effect produced by it. We have now to consider motion or change of motion in relation to the force which causes it and to the mass in which it occurs.

44. Newton's Laws of Motion.—In the latter part of the 17th. Century **Sir Isaac Newton,** an English philosopher enunciated three fundamental laws on force and motion on which the whole of dynamics is based. These laws do not admit of direct proof but are more or less of an axiomatic nature. They are, however, capable of explaining in every detail all observed facts relating to mechanics and are, therefore, accepted to be true. For instance, the motion of all heavenly bodies across the sky can be successfully explained by these laws and calculations based on them enable astronomers to make predictions so accurately that their validity can hardly be ignored.

45. The First Law of Motion.—*Every body continues in its state of rest or of uniform motion in a straight line except in so far as it is compelled by external impressed forces to change that state.*

The law consists of two distinct parts. The first

part of the law states that any body or a piece of matter is inert, *i.e.*, is unable by its own action to change its own state of rest or of motion. If it be at rest, it must continue to be at rest; and if it be in motion, *i.e.*, if it be moving in a certain direction at a certain rate, it cannot, of itself, change its direction or the rate of motion. This property of matter is called **inertia** and hence the law is sometimes called the **law of inertia**.

That a body at rest continues to be at rest in the absence of external forces is self-evident. A book resting on the table remains at rest unless it is raised up or pushed sideways. A football placed on the ground cannot move of itself but remains stationary unless kicked off by some one to make it move. But the statement that if a body be in motion it continues to be in the state of uniform motion in a constant direction in the absence of any external force is not so self-evident at first sight. For instance, a football kicked off from its position does not move on uniformly in a fixed direction for ever. In fact it gradually comes to rest and probably changes direction during its flight. It seems at first sight, therefore, that this is opposed to the above statement. A little consideration, however, shows that one cannot isolate the ball from external forces, such as the friction of the ground, that of the air, the gravitational pull of the earth and the force of the wind. It is these forces which retard the forward motion of the ball and possibly cause it to change its direction. If the wind does not blow and the ball moves in air the direction of motion does not appreciably change laterally. If, in addition, the earth's pull were absent and the space above the ground were complete vacuum there would have been no retardation and the ball would move on uniformly with the velocity imparted to it by the kick. If the ball moves along the surface of the earth an additional

force due to the friction of earth comes into play and it stops more quickly. We see, therefore, that the presence of external influences, not at once apparent, always impedes the uniform rectilinear motion of bodies which would otherwise continue for ever in their absence.

Numerous illustrations of the first law of motion are available in our daily life. When a motor bus suddenly starts off, a passenger standing on its floor is thrown backwards. This is because the body of the passenger is at rest when the bus is at rest. When the latter starts, the feet of the passenger which are in immediate contact with the floor of the bus share its motion while the upper part of his body tends to continue in its previous state of rest, thus resulting in the backward fall. If a coin is placed on a card which rests on the top of a tumbler glass and the card is quickly flicked away, the coin, due to inertia, remains stationary in its own position until the card moves away from beneath it and then drops into the glass.

Similarly, when a tram-car in motion suddenly stops, a passenger standing on its floor experiences a forward throw. For, when the car is moving, his whole body shares its motion. When it is suddenly stopped, the floor of the car and the feet of the passenger come to rest while the upper part of his body continues to move with the original velocity in the forward direction, thus throwing him in the direction of motion.

The second part of the law serves to define **force.** It states that the state of rest or of motion of a body will change only under a certain condition, *viz.*, when a force is impressed upon it. We may, therefore, define force as that *which produces or tends to produce a change of motion in a body on which it acts* (rest being regarded as motion with zero velocity) the

change of motion including any change in direction or of speed, or both.

It is to be remembered, in the interpretation of the law that when a body is not acted upon by a force, it must be in one of the two states of (1) being at rest, or (2) having a uniform motion in a straight line, either of which signifies that there is *no change* in the motion of the body. Further, the forces which bring about a change in the state of rest or of motion of a body are *impressed* from outside or external forces, and not *stresses* or internal forces between the parts of the body, for these latter cannot change the state of a body taken as a whole.

46. The Second Law of motion.—*The rate of change of momentum is proportional to the impressed motive force and the change of momentum takes place in the direction in which the force acts.*

This law introduces the idea of momentum or "quantity of motion" as it usually signifies and also provides with a quantitative definition of force.

Momentum.—If two balls of different masses are allowed to fall on the hand (supposed to be fixed in position) from a certain height, one after another, it will be seen that the heaveir ball strikes the hand with greater force than the lighter one. Evidently, the velocity acquired by both the balls (in falling through the same height) is the same (see § 26) so that the difference in the violence of impact must be due to the difference of masses.

If now the same ball is allowed to fall from two different heights it will be seen that the force with which it strikes the hand is greater when it is dropped from the higher level than when dropped from the lower level. Here, the mass is the same but the velocity acquired in the former case is greater than in the latter. Hence the difference in the violence of impact must be due to the difference of velocities.

We thus see that the violence of impact depends on two things, the mass and the velocity of the impinging body. In fact it is proportional to the product of the two. This product is called the momentum. We, therefore, define **momentum** of a body as *that property which it possesses by virtue of its mass and its velocity and which is measured by the product of these two quantities.*

Hence a change of momentum implies a change of mass or a change of velocity or both. But the mass of a body is always constant, so that any change of momentum of a moving body is only due to its change of velocity and, therefore, we can write

Momentum = mass × velocity

and Rate of change of momentum

= mass × rate of change of velocity

= mass × acceleration.

Since the momentum of a body depends on its velocity, it must be a vector quantity. A body moving due north with a certain velocity has a momentum equal and *opposite* to that which it possesses when moving due south with the same velocity. The *unit of momentum* is the momentum possessed by unit mass moving with unit velocity.

Quantitative Definition of Force—Let a body of mass m be moving with an initial vlocity u and let a force P act on it in the direction of its motion. The action of the force will be to change its momentum, *i.e.*, its velocity, so that after a time t its velocity will have some new value v. We have then

Change of momentum in time t $= mv - mu$

$\therefore$ Rate of change of momentum $= \dfrac{mv - mu}{t}$

By the law,

P is proportional to $m \times \frac{v-u}{t}$

i.e., $P \propto mf$

for $f = \frac{v-u}{t}$, from eqn. (4)

Or $P = k\,mf$ ... ... (33)

Hitherto we have measured the force P in any arbitrary undefined unit. But if we define the unit of force as that which when acting on unit mass produces unit acceleration we have $P=1$, when $m=1$ and $f=1$, so that from eqn. (33), k becomes equal to 1. We can then write

$P = mf$ ... ... (34)

which is known as the *Fundamental Kinetic Equation.*

Units of Force—The unit of force in the F. P. S. system of units is that force which produces an acceleration of 1 ft. per sec. per sec. in a mass of 1 lb. This unit is given the special name of a **poundal.**

The unit of force in the C. G. S. system of units is that force which produces an acceleration of 1 cm. per sec. per sec. in a mass of one gramme. This unit is called the **dyne.**

The above units of force are called **dynamical** or **absolute units** for they are absolute constants. There is another system of units of force called the **gravitational units,** in which a unit of force is the force with which unit mass is attracted by the earth towards it, *i.e.*, is the *weight* of a unit mass.

In the F. P. S. system, the gravitational unit of force is the weight of a pound of matter or a **pound-weight** (lb. wt.).

In the C. G. S. system, the gravitational unit of

force is the weight of a gramme of matter or a **gramme-weight** (gm. wt.).

Evidently the gravitational unit of force varies, though slightly, from place to place, for the weight of a body varies from place to place (see § 57). Hence the gravitational units are used only for practical purposes where the slight variation does not matter much ; thus engineers speak of forces in terms of tons weight or kilograms weight.

But for scientific purposes it is necessary that a unit should be an absolute constant. Hence in scientific work the dyne or the poundal is always used as the unit of force.

RELATION BETWEEN THE DIFFERENT UNITS

A force of 1 poundal acting on a mass of 1 lb. produces an acceleration of 1 ft. per sec. per sec.

Now 1 lb. = 453·6 gms.
and 1 ft. = 30·48 cms.

∴ A force of 1 poundal acting on a mass of 453·6 gms. produces an acceleration of 30·48 cms. per. sec. per sec.

i.e., a force of 1 poundal acting on a mass of 1 gm. produces an acceleration of $30·48 \times 453·6$ cms. per sec. per sec.

But a force of 1 dyne acting on a mass of 1 gm. produces an acceleration of 1 cm. per sec. per sec.

∴ 1 poundal $= 453·6 \times 30·48$ dynes
$= 13825·7$ dynes

Dimension of Force.—Since the dimensions of an acceleration are $[L]\,[T]^{-2}$ (§.27), therefore the dimensions of a force are, from eqn. (14), $[M]\,[L]\,[T]^{-2}$.

Principle of Physical Independence of Forces.—From the latter part of the law it is understood that if there are two or more forces acting simultaneously on the same body, each force produces the same effect as if the other forces were not acting. This is a fundamental principle of mechanical science and is known as the *Principle Physical Independence of Forces*. This gives us, for the composition of two or more forces, the well-known law of the *Parallelogram of Forces*, which is, as has already been seen, exactly similar to that of the Parallelogrm of Velocities.

47. The Third Law of Motion—*To every action there is an equal and opposite reaction.*

The terms 'action' and 'reaction' in this law apply to forces. Here *action* means the force which one body A exerts on another. The law implies that the second body B always exerts on the first a force in the same straight line, equal to the former in magnitude, and opposite in direction. This force is called the *reaction* of the second body *on* the first. This mutual action between two bodies is generally called a *stress*; actions and reactions are thus merely the opposite aspects of the stress between two bodies. The law is, therefore, called the **law of reaction** or the law of stress.

To quote Newton's own illustration.—"If a man presses a stone with his finger, his finger also is pressed by the stone; in other words, the stone resists. Here the pressure exerted by the finger on the stone is an action; and the equal and opposite pressure exerted by the stone on the finger is the reaction."

Similarly, when a book rests on a table, it exerts a pressure (which is its weight), vertically *downwards* on the table; at the same time, the table resists or exerts a pressure vertically *upwards* on the book. The

former is the action, while the latter is the reaction. Since the book is at rest, the forces, *i.e.*, action and reaction must be exactly equal and opposite to each other.

Again, when a ladder is resting against a wall, it presses against the wall which, if not sufficiently strong, may actually be overturned. The wall also exerts an equal force against the ladder, which prevents the ladder from falling over, as it would certainly have done, had there been no support in the same position.

It is evident from what has been said above that a force cannot exist by itself. It cannot act in any way whatsoever unless it calls into play an opposing reaction ; the magnitude of a force that can be exerted on a body is limited by the amount of reaction which the body can itself exert. One cannot, for example, support one ton weight on his umbrella.

It is important to note that the law holds equally true whether the reacting bodies are at rest or in motion.

The truth of this law is apparent in the case of objects which are at rest, as for example, a person standing on the floor or a boy trying in vain to move a standing motor car. But in the case of objects in motion the truth of the law is not so self-evident. Let us take the case of a horse pulling a cart. So long as the system does not move it is quite easy to see that the forward pull exerted by the horse along the traces on the cart is balanced by the backward pull of the cart exerted on the horse via the traces. But when the system is set in motion, difficulty arises. It is sometimes argued that since the horse pulls the cart, the forward force exerted by the horse on the cart along the traces must be greater than the backward force exerted along it by the cart on the horse. But this is not true. The forward and back-

ward forces acting along the traces are exactly equal. How does then motion set in ? The answer to this question can be best obtained if we consider the horse and the cart separately.

Let us consider the cart first. The forces acting on it are (Fig. 41),

(*a*) its weight acting vertically downwards

(*b*) the upward thrust of the ground

(*c*) the resistive force R due to friction acting backwards

(*d*) the forward pull T acting along the traces.

Of these, forces (*a*) and (*b*) balance each other. *The cart moves when* T>R.

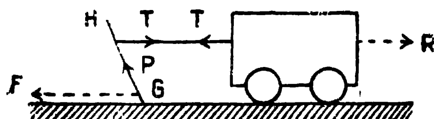


Fig. 41
Horse pulling a cart.

The forces acting on the horse are,

(*a*) its weight acting vertically downwards

(*b*) the vertical component of the thrust P of the ground which latter acts in the direction GH.

(*c*) the horizontal component F of the thrust

(*d*) the backward pull T.

Of these, forces (*a*) and (*b*) balance each other. The horse moves when F>T. So that although the backward pull on the horse along the traces is equal to the forward pull on the cart along the traces, there is no reason why the system cannot move, the only conditions for motion being T>R and F>T.

48. Uniform Motion in a Circle; the Normal Force. If a body moves round a circle with a uniform speed, it passes over equal arcs in equal times. It is to be noted, as has already been stated, that though the speed remains constant, the direction of motion and hence the velocity is always changing, acting in a tangential direction at any point on the circular path. We have, from art. 33, the relation $\omega = v/r$, where ω is the angular velocity of the body about the centre, v, the uniform speed and r, the radius of the circle.

A body once put in motion in any direction tends, owing to its inertia, to move always in that direction with its velocity unchanged. Hence whenever a body is seen to move in a curved path, there must be some force urging it always towards the centre of curvature of the curve. This force is called the **normal force.**

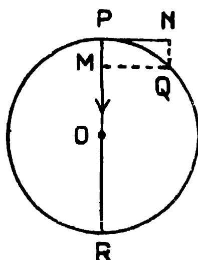


Fig. 42
The normal force in circular motion

Expt. 2. Tie a piece of stone to a string and swing it round in a circle by holding the other end. It will be felt that in order to keep the stone in its circular path, a force is to be exerted by the hand along the string towards the centre of the path. If the string is cut off or let go, the stone ceases to be acted upon by the tension of the string and tends to fly off in a direction tangential to its circular path at the point where the stone was when the force towards the centre was made to vanish.

Magnitude of the Normal Force:—In § 34, we have seen that if a body moves *uniformly* in a circle of radius r with a speed v, then the normal acceleration with which the body always tends to move towards the centre of the circle is given by

$$f = \qquad (35)$$

Now if the mass of the body be m, then the normal force urging it towards the centre is given by, from eqn. [34],

$$R = \frac{mv^2}{r} \qquad \ldots \qquad \ldots \qquad (36)$$

49. Centripetal and Centrifugal Forces.—The normal or the deflecting force, spoken of in the last article, which acts *upon* the rotating body in a direction *towards* the centre of curvature in order to constrain it to move in a curved path, is called the **centripetal force** and is equal to $\frac{mv^2}{r}$

Since every action is accompanied by a reaction, equal and opposite to it (Newton's third law of motion), there must also be an equal force directed *away from the centre*; this force is called the **centrifugal force** and is also equal to $\frac{mv^2}{r}$. It is a force exerted *by* the rotating body *on* the centre of curvature. Thus in the case of the stone in the string in Expt. 2 it is the force exerted *by* the stone along the string and *not* a force acting upon the stone. It is due to this force that the string is kept tight against the centre of curvature. It is to be noted that if, in any case, the centripetal force vanishes, its reaction *viz.*, the centrifugal force does also vanish at once.

Illustrations of the influence of centrifugal force are ample. When a cyclist turns round a corner he invariably leans to the inside of the curve. (Fig. 43). Let the speed with which a cyclist is rounding a curve of radius of curvature r be v. Then the cen-

trifugal force with which he is pulled away from the centre of curvature is $\frac{mv^2}{r}$, where m is the combined mass of the cycle and the cyclist. To prevent a fall this force must be counterbalanced. This is done by the resultant of the only two forces acting on the system, *viz.*,

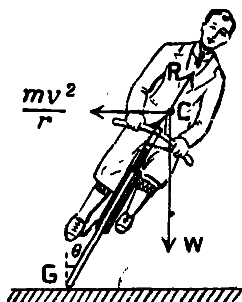


Fig. 43
Inclined position of a cyclist when turning round a curve.

(i) the reaction R of the ground acting in the direction GC

(ii) the weight W acting vertically downwards.

Let the triangle ABC in fig. 44. represent the forces, AB representing the weight, BC the reaction of the ground and CA the centrifugal force which is parallel to the plane of the track (horizontal in this particular case).

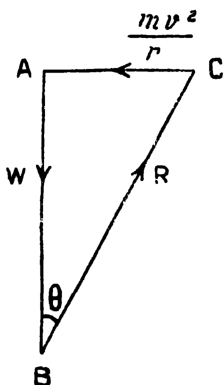


Fig. 44.
Triangle representing the forces in Fig. 43.

We then have,

$$\tan\theta = \frac{AC}{AB} = \frac{mv^2}{r} / W = \frac{v^2}{gr}$$

for $W = mg$ (see 56), g being the acceleration due to gravity.

Thus we see that the greater the speed and the sharper the curve (*i.e.*, the smaller the radius of curvature r) the greater must the cyclist be inclined to the vertical.

The road-bed at a bend in a railway line is often

inclined, so that the outer rail is a little higher than the inner one. When a train rounds a curve, a part of its weight and the reaction at the flanges balance the centrifugal force, called into play, when the train moves in the curved path. Moreover, the action is helped by reducing the speed of the train at the bend.

For a similar reason, a horse and its rider while rounding the track in a circus always incline their bodies inward, and the greater their speed, the greater is this inclination.

In looping the loop, or in the toy, called the *Centrifugal Railway*, the momentum acquired by the car in falling through a height and the reaction at

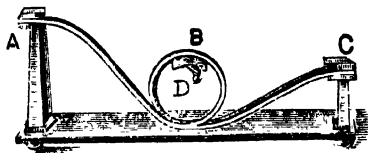


Fig 45
The Centrifugal Railway

each point of the loop exerted against the car enable it to round the bend. Similarly, if a small can of water hung by the handle is rapidly swung round in a vertical circle, no water will come out, even when the bottom is uppermost.

Governors.—We have seen that the centrifugal force tends a rotating body to fly away from the centre and is given by

$$\frac{mv^2}{r} = mrw^2 \quad \ldots \quad \ldots (37)$$

Hence if the speed of rotation be increased the tendency to fly off will also increase thus urging.

the body to rotate in a larger circle. If the speed be decreased the body will tend to rotate in a smaller circle. This properly is utilised in what we call governors, *i.e.*, appliances which automatically cortrol a motion.

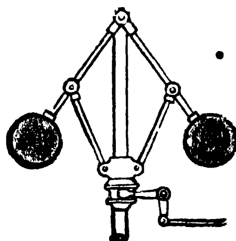

Fig. 46
Watt's Governor

Fig 46 represents a portion of *Watt's Governor* which is used to regulate the supply of steam in some forms of steam-engines. Two heavy balls are fastened to arms hinged at the top on a vertical axis which is connected with main shaft of the engine to share its rotation. The mechanism for opening and closing the steam-port so as to vary the supply of steam is connected with the balls. When the engine runs too fast, the balls rotate in wider circles and hence rise up and partially cut off the supply of steam , when the speed is too slow, the balls fall and open the port. In this way, the engine automatically controls its own speed.

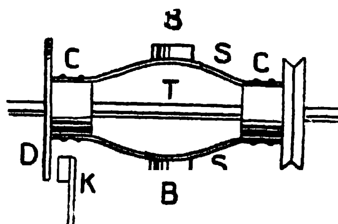


Fig. 47
Gramophone Governor

Another type of governor is used in gramophones.

to control the speed of revolution of the turn-table. It consists of three or four thin elastic strips S of metal fixed at their extremities on two collars C (Fig. 47). One of these collars is fixed on a spindle, T which shares the rotation of the turn-table. The other collar can slide freely along the spindle. A small metal bob B is fixed at the centre of each strip. A disc D attached to the movable collar presses against a brake-block K held in position by a spring. The clock-work device of the gramophone tends to rotate the spindle with the maximum possible speed but the brake block pressing against the disc puts a limit to the fly off of the metal bobs thus keeping the speed down to any desired value.

51. Flattening of the Earth.—The flattening of the earth is believed to be caused by the action of centrifugal forces called into play by the rotation of the earth about its own axis. The earth, as is held by geologists, was once in a molten state. As it rotated about its axis, the materials composing the earth yielded to the centrifugal forces, which are greatest at the equatorial regions and negligibly small near the poles. The result was that the earth has become a spheroid having a bulging at the equator and consequent flattening at the poles (Fig. 48).

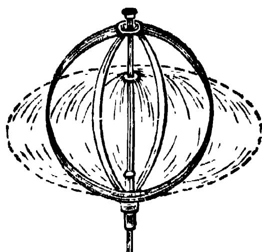


Fig. 48.
Illustrating the flattening of the earth.

Fig. 48 represents a model to illustrate the flattening of the earth by rotation. It consists of a central rod that carries several thin elastic strips of metal in the form of a circular hoop. The hoop is fixed at the bottom and is attached to a

collar at the top which can slide up and down the rod.

If the rod is rapidly rotated, it is seen that the hoop is bent from its circular form and becomes flattened at the top and the bottom and bulged at the centre.

Exereise IV.

1. What do you mean by inertia ? Distinguish between inertia of rest and inertia of motion. Give examples.

2. State and explain Newton's First Law of Motion and show how a definition of force can be derived from it.

3. State and explain Newton's Laws of Motion. Show how a quantitative measure of force may be derived from the Second Law.

4. State Newton's Third Law of Motion. How does it hold in the case of a horse pulling a cart ?

5. What is meant by a dyne and a poundal ? Find the relation between them.

6. A force of 250 dynes acting on a body for one second imparts to it a velocity of 10 cm./sec. Find the mass of the body. [Ans.—25 gms.]

7. A constant force acting on a mass of 20 lbs. for 2 secs. gives it a velocity of 8 ft./1 sec. Find the magnitude of the force. [Ans.—80 poundals]

8. A railway train running at the rate of 50 miles per hour stops completely in 5 mins. If the mass of train is 10 tons, find the magnitude of the retarding force, assuming it to be constant. [Ans.—13440000 poundals]

9. What are centrifugal and centripetal forces ? Find the magnitude of the centrifugal force when a stone weighing 2 lbs. is swung round a circle of radius 10 ft. with a linear speed of 30 ft /sec. [Ans.—180 poundals]

10. A cyclist running at the rate of 48 ft./sec. turns round the corner of a track of radius of curvature 720 ft. Assuming $g=32$, find the inclination which the cyclist must make with the vertical so that he may run without falling. [Ans.—45°]

PART VI

GRAVITATION—FALLING BODIES

CHAPTER VI

GRAVITATION—FALLING BODIES

52. Gravity.—It is a matter of everyday experience that whenever a heavy body is unsupported, it falls to the ground. And as it falls, its velocity is gradually increased ; in other words, the motion is accelerated.

Since there is an acceleration or a change of velocity in the downward direction, it follows from Newton's second law of motion that there is a force in this direction. Newton discovered that this force is due to the attraction of the earth on the body. This attractive force of the earth is called **gravity,** and the acceleration produced by it is called **acceleration due to gravity,** or simply the **acceleration of gravity** and is always denoted by the letter g.

53. The Law of Gravitation.—The action of gravity is an illustration of and at the same time a particular case of the **law of universal gravitation** which was first stated by Newton as given below :

Every particle of matter in the Universe attracts every other particle, along the line joining them, with a force which varies directly as the product of their masses and inversely as the square of the distance between them.

This force of attraction is called **gravitation.**

It follows from this law that two bodies of masses m and m' separated by a distance d attract each other with a force F which is given by

$$F = G.\frac{mm'}{d^2} \quad \dots \quad \dots \quad (38)$$

Where G is the constant of proportionality and is known as the **gravitational constant.** The value of G is $6{\cdot}64 \times 10^{-8}$ C. G. S. units, as obtained by **C. V. Boys** in 1895. If the masses of two bodies be known and also their distance apart, the force of attraction acting between them can then be calculated.

EXAMPLE :—

1. Two lead balls, each weighing 10 lbs, are suspended by means of strings so that they are 1ft. apart. Find the force of attraction exerted between them ($G = 6{\cdot}6 \times 10^{-8}$ C. G. S. units).

Since G is given in C. G. S. units, the masses and distance should also be converted into C. G. S. units. Thus we have,

$$m = m' = 10 \text{ lbs.} = 453{\cdot}6 \times 10 = 4536 \text{ gms.}$$

$$d = 1\text{ft.} = 30{\cdot}48 \text{ cms.}$$

Hence the force of attraction F is given by

$$F = G\frac{mm'}{d^2}$$

$$= 6{\cdot}6 \times 10^{-8} \cdot \frac{4536 \times 4536}{30{\cdot}48^2}$$

$$= 1{\cdot}462 \times 10^{-3} \textit{ dyne.}$$

This force is too small to produce an appreciable effect in any practical case. It is only with very delicate instruments that the effect of attraction in such cases can be detected and measured.

54. Attraction due to Gravity.—In order to calculate the attraction exerted by the earth on a body, the whole mass of the earth is taken to be concentrated at its centre (see § 67) and the body is considered as a particle because it is generally very small in comparison with the size of the earth. If a body of mass *m* be situated at a height *h* above the

surface of the ground (Fig. 49) and if M and R be the mass and radius of the earth, then the force of attraction F between the two is given by

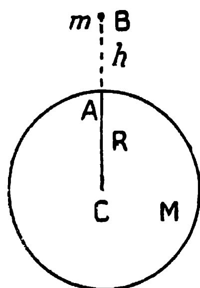


Fig. 49.
The earth attracting a body

$$F = G.\frac{Mm}{(AC+AB)^2}$$

$$=G.\frac{Mm}{(R+h)^2} \qquad (39)$$

When the body is within a few hundred metres above the ground, as is usually the case with experiments on falling bodies, h is negligible in comparison with R which has a mean value of *km*. We can write, in such cases,

$$F = G.\frac{Mm}{R^2} \qquad \ldots \qquad \ldots (40)$$

The earth and the body will attract each other with this force. But since the mass of the earth is very great in comparison with that of any terrestrial object, this mutual attraction will cause the body, when unsupported, to move towards the centre of the earth along the line joining the latter to the body. The earth will not move for, its mass being very great, the acceleration produced in it by the attractive force is extremely small.

Weight of a Body—When a body is raised above the ground, a downward force is experienced. This is due to the gravitational attraction exerted by the earth on the body and is commonly known as its **weight.** Thus we may define *the weight of a body as the force with which the body is attracted by the earth towards its centre.*

We know that the vertical direction at any point

on the earth is the direction of the line joining the centre of the earth to that point, *i.e.*, the radius of the earth at that point. It, therefore, follows that the weight of a body always acts vertically downwards. This explains why a plumb line always directs vertically.

55. Acceleration of Gravity is the same for all Bodies at a Given Place—Since the attractive force exerted by the earth on a body of mass m, situated at a small height above the ground, is given by G $\frac{Mm}{R^2}$ [eqn. (40)] it follows, from Newton's second law of motion, that

$$\text{acceleration produced on the body} = \frac{\text{attractive force exerted on the body}}{\text{mass of the body}}$$

$$= \frac{G.\frac{Mm}{R^2}}{m}$$

$$= \frac{G.M}{R^2}$$

But the acceleration produced by the earth's attraction, *i.e.*, the acceleration due to gravity is denoted by g. we may therefore write

$$g = \frac{GM}{R^2} \qquad \ldots \qquad \ldots [41].$$

Now G and M are constants and if we consider a particular place on the earth, R is also constant. Thus it is seen that the acceleraion due to gravity is a constant, *i.e.*, is the same for all bodies at the same place, irrespective of their masses.

We also have from Newton's second law

$$W = mg \ldots \qquad \ldots \qquad \ldots (42)$$

where W is the earth's attractive force on the

body. *i. e.*, its weight. Since g is a constant at a particular place, it follows from the above equation that *the weight of a body is proportional to its mass.*

56. Variation of the Acceleration due to Gravity—We have seen above that the acceleration due to gravity is the same for all bodies at a particular place. Due to the peculiar shape of the earth which is flattened at the poles, the length of its radius varies from point to point. In fact it is about 13·4 miles smaller at the poles than at the equator. Hence it follows, from eqn. (41), that the value of g is greater at the poles than at the equator.

If, in addition, we consider a point at a height above the surface of the earth comparable to the radius of the earth, we cannot neglect h in eqn. 39 and then g is given by

$$g = \frac{G.\ M,}{(R+h)^2}$$

Hence it is evident that the acceleration of a freely falling body at a considerable height above a point on the surface of the earth is less than its value at that point and that its value decreases as the height is increased.

EXAMPLE :—

Assuming the mass of the earth to be $5{\cdot}9 \times 10^{27}$ gms., the equatorial radius 6378 kms, polar radius 6357 kms. and G, $6{\cdot}64 \times 10.^{-8}$ C. G. S. units, calculate the value of g at (i) the poles, (ii) the equator, (iii) a height of 1000 kms. above the equator and (iv) a height of 1000 kms. above the poles.

$$\text{(i)} \quad g = \frac{GM}{R^2} = \frac{6{\cdot}64 \times 10^{-8} \times 5{\cdot}98 \times 10^{27}}{(6357 \times 10^5)^2}$$

$$= 985{\cdot}4\ cm/sec.^2$$

(ii) $$g = \frac{GM}{R^2} = \frac{6{\cdot}64 \times 10^{-8} \times 5{\cdot}98 \times 10^{27}}{(6378 \times 10^5)^2}$$

$$= 979{\cdot}5 \ cm/sec.^2$$

(iii) $$g = \frac{GM}{(R+h)^2} = \frac{6{\cdot}64 \times 10^{-8} \times 5{\cdot}98 \times 10^{27}}{(7378 \times 10^5)^2}$$

$$= 682{\cdot}9 \ cm/sec.^2$$

(iv) $$g = \frac{GM}{(R+h)^2} = \frac{6{\cdot}64 \times 10^{-8} \times 5{\cdot}98 \times 10^{27}}{(7357 \times 10^5)^2}$$

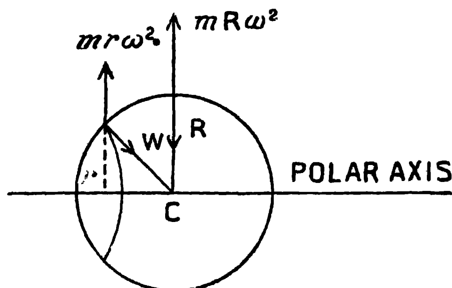
$$= 735{\cdot}8 \ cm/sec.^2$$

The observed values of g on the surface of the earth at the equator and at the poles are respectively 979·03 and 983·4 cm/ sec^2. The mean value of g is taken to be 32·2 ft. per sec. per sec. or 981 cms. per sec. per sec.

57. Variation of the Weight of a Body.—Since the weight W of a body is given by (eqn. 42) $W = mg$, it is evident that as the value of g is different at different places on the surface of the earth and at different levels above it, the weight of a body also varies in the same way. It is minimum at the equator and maximum at the poles and as the body is raised higher and higher up in space it decreases continually until it becomes inappreciable.

There is another cause on account of which the weight of a body on the surface of the earth increases with increasing latitude. This is due to the rotation of the earth about its axis. Owing to the diurnal rotation of the earth, every body on its surface revolves and is therefore subjected to a centrifugal force. If ω be the angular velocity of the earth and r be the radius of the circle in which a body on the surface of the earth rotates (Fig. 50) then from

eqn. (36), the centrifugal force is given by $mr\omega^2$ where m is the mass of the body.



Fig, 50.
Variation of weight of a body due to earth's diurnal rotation.

A part of this force (the resolved part along the earth's radius through the body) opposes the true weight W of the body, thus making it appear lighter than what it would be in the absence of the earth's rotation. This opposing force decreases with increase of latitude for two reasons. First, the centrifugal force diminishes with increase of latitude for the value r diminishes as we move towards the poles. It is greatest at the equator and vanishes altogether at the poles. Secondly, the portion of the centrifugal force which opposes the attractive force of the earth directed towards its centre, that is the true weight of a body, decreases as the latitude is increased. At the equator, the whole of the centrifugal force opposes the weight of the body. At points near the poles, only a minute fraction of the centrifugal force is effective in opposing the true weight of the body. Thus what we ordinarily call the weight of a body is not its true weight but its *apparent* or effective weight.

58. Relation between Absolute and Gravitational units of force.—We have already seen (see §. 46) that the gravitational unit of force is the weight of unit mass, *i.e.*, is the force with which unit mass is attracted towards the earth. This force produces in it g units of acceleration. But an absolute or dynamical unit of force produces unit acceleration in unit mass. It follows therefore that

1 gravitational unit $= g$ absolute units. Hence, in the F. P. S. system,

1 lb. wt. $= g$ poundals $= 32{\cdot}2$ poundals.

And in the C. G. S. system,

1 gm. wt. $= g$ dynes $= 981$ dynes.

Since the value of g varies from place to place the magnitude of the gravitational unit of force also varies.

59. The Guinea and Feather Experinent.—Since the acceleration due to gravity is the same for all bodies at a given place, it follows that every body, heavy or light, must fall, when unsupported with equal rapidity.

But this is not in conformity with our common experience, as we are accustomed to see light bodies such as feather, pieces of paper, etc., fall very slowly to the ground, while a heavy body let fall from the same height reaches the ground very quickly. The difference in the rapidity of fall was formerly thought to be inherent in the nature of the materials of bodies.

Previous to GALILEO'S time men believed in ARISTOTLE (about 357 B. C.) who had said that a ten-pound weight would fall ten times as fast as a one-pound weight. Galileo, in the latter half of the sixteenth century, argued that two such weights, dropped from the same heights, would take exactly the same time in falling to the ground. In the pre-

sence of the professors and students of the University of Pisa he dropped balls of different sizes and materials from the top of the *Leaning Tower of Pisa*, 180 ft. high, and showed that they fell in precisely the same time. He held that all bodies even the lightest would fall at the same rate, but for the resistance offered by the air to impede the motion of bodies, the resistance increasing with the extent of surface exposed by the body. Thus the effect is more marked, the less the mass of a body and the greater the surface it presents. If, by some means or other, this air-resistance be eliminated, all bodies, heavy or light, would take the same time to fall through a given height. Modern Physics may be said to begin with Galileo who discovered the laws of falling bodies and the laws of the pendulum.

Galileo's conclusion that in a vacuum all bodies would fall with the same velocity,—could not, however, be put to test by experiments in his time, the air-pump not having yet been invented. After its invention sixty years later (1650) by OTTO VON GUERICKE, the experiment was performed by NEWTON and is now well known as the '*Guinea and Feather*' experiment.

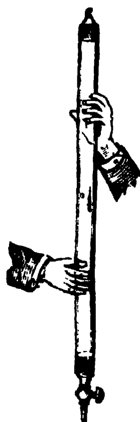


FIG. 51

Guinea and Feather experiment.

Expt. 3. Take a large coin or a disc of metal of about 2″ diameter and cut a piece of paper slightly smaller than the coin. Hold these side by side at the same height above the table and drop them simultaneously. The metal disc touches the table first and then does the paper.

Now lay the paper on the top of the metal and drop them carefully. Both reach the table simultaneously. Here the metal disc overcomes the resistance of the air which would otherwise retard the motion of the paper.

Repeat the experiment by placing the paper on the metal

disc such that part of its surface is exposed to the air. Now the paper will be left behind but not to the same extent as in the first case.

Expt. 4. Fig 51 represents a stout glass tube about a metre long, closed by a cap screwed to one end, and is provided with a stop-cock at the other. Introduce a coin and a piece of paper or a feather into the tube. Exhaust the inside of the tube by means of an air-pump. After closing the stop-cock, disconnect the tube from the pump.

Invert the tube suddenly : the coin and the paper will strike the bottom simultaneously. Repeat this.

Now introduce a little air within the tube by opening the stop-cock for a moment. Invert the tube again : now the feather becomes slightly retarded in its fall.

60. Experimental Determination of g **:—** There are several methods of determining the value of g. The Inclined Plane and the Atwood's Machine are employed for this purpose but the value of g obtained with these instruments is only approximate. The most accurate determination of g is, however, made by means of experiments with the pendulum which will be discussed later on (§. 94).

61. Laws of falling bodies.—As a body, when unsupported, falls freely with an uniform acceleration g, all problems concerning falling bodies may be solved, by the formulæ established in art. 28 in which g is to be substituted for f, the resistance of the air being neglected.

If a body starts from rest, being simply allowed to fall from a height, we have for its velocity at time t

$$v = gt \qquad \ldots \qquad \ldots (42)$$

and for the space traversed

$$h = \tfrac{1}{2}gt^2 \qquad \ldots \qquad \ldots (43)$$

From (42) & (43) $v^2 = 2gh \qquad \ldots \qquad \ldots (44)$

From these equations and from art. 55 we may state the following laws of falling bodies in vacuo, *i.e.*, when they experience no resistance :—

(*1*). *In a vacuum all bodies fall with equal rapidity.*

(*2*). *The velocity acquired by a falling body is proportional to the duration of its fall i.e.,* $v \propto t$. Thus if the velocity at the end of a second is 32 ft. per sec., that at the end of two seconds is 64 ft. per sec., at the end of 3 seconds is 96 ft. per sec., and so on.

(*3*). *The space traversed by a falling body in a given time reckoned from the start is proportional to the square of that time, i.e.,* $h \propto t^2$. Thus if a stone falls through 16 ft. in one second, it will drop through 64 ft. in two seconds, 144 ft. in 3 seconds and so on.

62. Fall of Bodies Projected downwards or upwards.—If a body be projected downwards with an initial velocity u, the formulæ of art. 29 become

$$v = u + gt \quad \ldots \quad \ldots \quad (45)$$

$$h = ut + \tfrac{1}{2}gt^2 \quad \ldots \quad (46)$$

$$v^2 = u^2 + 2gh \quad \ldots \quad (47)$$

If a body be projected upwards with a given upward velocity u, we are to substitute $-g$ for f, since the acceleration due to gravity is opposing the ascent. The upward direction of u being taken as positive, the formulæ now become

$$v = u - gt \quad \ldots \quad \ldots \quad (48)$$

$$h = ut - \tfrac{1}{2}gt^2 \quad \ldots \quad \ldots \quad (49)$$

$$v^2 = u^2 - 2gh \quad \ldots \quad \ldots \quad (50)$$

From these formulæ any problem on the motion of falling bodies may be solved. For example, *the greatest height h*, to which a body rises when projected upwards with a velocity u, is given by

$$o = u^2 - 2gh$$

for $v = o$, when the body just ceases rising.

$$h = u^2/2g \quad \ldots \quad \ldots \quad (51)$$

Similarly, *the time to attain this greatest height* is given by

$$v = o = u - gt$$
$$\therefore \quad t = u/g \qquad \ldots \quad (52)$$

Again, after reaching its greatest height the body will begin to fall and when it returns to its point of projection, the final height becomes zero. *The whole time of flight* may be obtained by putting $h = o$ *and* $f = -g$; we thus have

$$\begin{aligned} 0 &= ut - \tfrac{1}{2}gt^2 \\ &= t(u - \tfrac{1}{2}gt) \end{aligned}$$

Discarding the solution $t = 0$, we get

$$u = \tfrac{1}{2}gt \quad \text{or} \quad t = 2u/g \qquad \ldots \ (53)$$

Equations (52) and (53) show that the whole time of flight is double the time taken by the body to reach the greatest height. In other words, the *time taken during the ascent is equal to the time taken during the descent.*

63. Flight of a Projectile.—Let us suppose that a stone is thrown horizontally with a velocity u from the top of a cliff. Due to this horizontal velocity, the stone will cover a horizontal distance x in time t given by

$$x = ut \qquad \ldots \qquad \ldots \quad (a)$$

Due to the force of gravity it will fall through a vertical distance y in the same time t given by

$$y = \tfrac{1}{2}gt^2 \qquad \ldots \qquad \ldots \quad (b)$$

Eliminating t, we get

$$y = \frac{g}{2u^2} \cdot x^2 \qquad \ldots \ (54)$$

This equation expresses the relation between the horizontal and vertical distances traversed by the projectile at any instant and hence defines the

path of the projectile. It represents a parabola with its axis vertical and its apex at the point of projection.

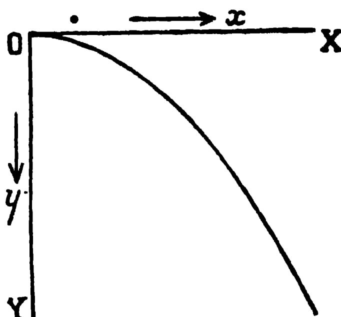


Fig. 52
Path of a projectile

If the height of the cliff be h, then the time taken by the projectile to reach the ground is given by

$$t = \sqrt{\frac{2h}{g}}$$

and the horizontal distance d from the foot of the cliff at which the projectile will strike the ground is given by

$$d = u\sqrt{\frac{2h}{g}}.$$

If a body be projected upwards with a velocity u in a direction inclined to the horizontal by an angle θ, the horizontal and vertical components of this velocity are $u \cos \theta$ and $u \operatorname{Sin} \theta$ respectively (cf. §31).

After time t, the vertical velocity of the projectile is, by eqn. (48),

$$u \operatorname{Sin} \theta - gt$$

Consequently it will be zero, and the projectile will have reached its highest point when

$$t = \frac{u \text{ Sin } \theta}{g}$$

And since time taken by the projectile to reach the highest point is equal to the time taken to descend

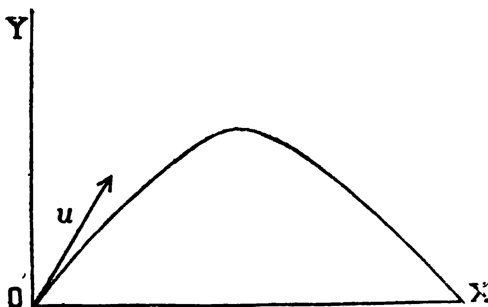


Fig 53
Range of a projecti'e

from this point to the ground (§. 63), the whole time of flight is given by

$$\frac{2u \text{ Sin } \theta}{g}$$

During the whole time of flight the horizontal component of the velocity has its initial value $u \cos \theta$.

Hence the horizontal distance which the projectile will cover before it strikes the ground, *i.e.*, the *horizontal range*, is given by

$$u \cos \theta \times \frac{2u \sin \theta}{g} = \frac{2u^2 \sin \theta \cos \theta}{g} = \frac{u^2 \sin 2\theta}{g}$$

EXAMPLES :—

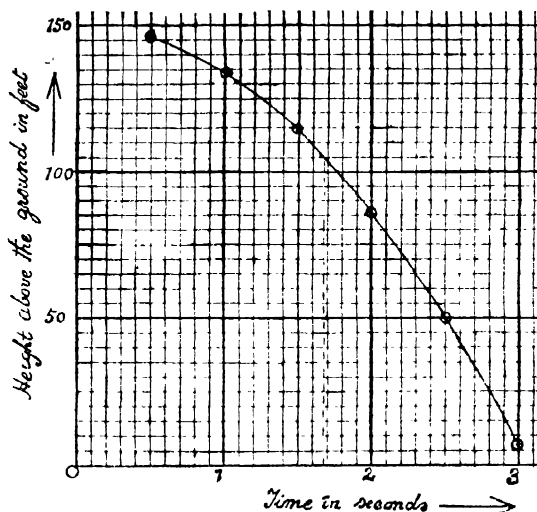
1. Draw a curve on the squared paper supplied to indicate the heights above ground, at interval of half of a second, of a body falling freely from rest at a height of 150 ft. Find from your graph the position of the particle after 1·67 seconds. [*C. U.—1912*]

The space traversed by a body in time t falling from rest can be obtained from the formula

$$h=\frac{1}{2}gt^2$$

Taking $g=32 ft.$ *per sec. per sec.* and calculating the distances fallen through at the end of every half second, the following table is prepared :

TIME IN SECONDS	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
HEIGHT FALLEN THROUGH	4	16	36	64	100	144
HEIGHT ABOVE GROUND IN FEET	146	134	114	86	50	6



In the graph given above
1 small division along x-axis=0.1 sec.
1 small division along y-axis=5 ft.

It is seen from the graph that at the time 1·67 seconds, the particle is at a height of 105 ft. above the ground.

2. If a heavy body is thrown vertically up to a given height and then falls back to the earth, show that, neglecting the resistance of the air, it passes each point of its path with the same velocity when rising and when falling.

Let u = velocity of projection
v = velocity at a given height h

$$\text{Then } v^2 = u^2 - 2gh$$
$$\therefore \quad v = \pm\sqrt{u^2 - 2gh}$$

which shows that the velocities are equal numerically, but opposite in sign, at any point of the path, when rising and when falling.

3. A ball is thrown vertically up and caught again in 6 seconds. Find the velocity of projection and the greatest height.

Let u = initial velocity and h = greatest height.

$$\text{From } s = ut + \tfrac{1}{2}ft^2 \quad \text{we get here}$$
$$o = u.6 - \tfrac{1}{2}.32.6^2$$
$$\text{whence } u = 96 \text{ ft. per sec.}$$
$$\text{And } v^2 = u^2 - 2fs \quad \text{gives}$$
$$u^2 = 2gh \quad \text{or} \quad h = \frac{96 \times 96}{2 \times 32} = 144\text{ft.}$$

Exercise V

1. A body falls freely for 6 seconds; find the distances it will fall through in the last second and in the whole time.

2. State the Laws of falling bodies and illustrate them by suitable examples.

Describe and explain the celebrated Guinea and Feather experiment. [*C. U.—1926*]

3. A cannon-ball is shot horizontally from the top of a tower 49 ft. high with a velocity of 200 ft. per second. Find at what distance from the tower the cannon ball will strike the ground.

4. A stone is let fall from the top of a railway-carriage which is travelling at the rate of 30 miles an hour. Find what horizontal distance and what vertical distance the stone will have passed through in one-tenth of a second.

5. The intensity of gravity at the surface of the planet Jupiter being about 2·6 times as great as it is at the surface of the earth, find approximately the time which a heavy body would take in falling from a height of 167 feet to the surface of Jupiter.

6. An arrow is shot vertically upwards with a velocity of 104 feet per second when it leaves the bow. How long will it be before it reaches the ground again ?

7. What is meant by 'acceleration of gravity' ? How do you prove that it varies from place to place on the earth's surface ? How does it vary ? (*C. U.—1933*)

8. A stone is dropped from a balloon at a height of 400 ft. above the ground and it reaches the ground in 6 seconds. Find the velocity with which the balloon was rising.

9. A stone dropped into a well reaches the water with a velocity of 80 ft. per second, and the sound of its striking the water is heard $\frac{27}{12}$ seconds after it is let fall. Find from these data the velocity of sound in air.

10. Explain what is meant by ***mass*** and ***weight*** and how they are measured.

A body is weighed at the surface of the earth, at sea-level and at the top of a mountain. State, in general terms, how the position will affect the weight and the mass of the body. Give reasons for your answer. (*C. U.—1920*)

11. State the laws governing the motion of bodies falling freely under the action of gravity.

Two stones are projected vertically upwards at the same instant. One ascends 112 ft. higher than the other, and returns to earth 2 seconds later. Find the velocities of projection of the stones [g=32 ft. per sec. per sec.] (C.U.—1935)

CHAPTER VII

CENTRE OF GRAVITY

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CENTRE OF GRAVITY

64. Centre of Parallel Forces.—Let a number of like parallel forces of magnitude P, Q, R, S, etc., act at points A, B, C, D, etc., of a body (Fig. 55). The resultant force of P and Q is a force of magnitude $(P+Q)$ acting at a point E in AB, such that $P \times AE = Q \times EB$, and is parallel to both of them. The forces $(P+Q)$ acting at E and R at C are equivalent

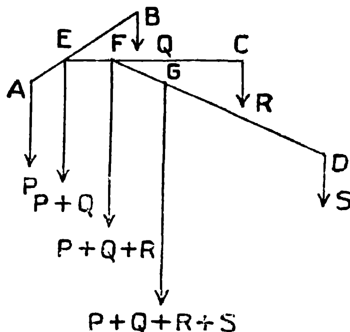


Fig. 55
Centre of parallel forces.

to a single resultant force $(P+Q+R)$ acting at F, parallel to them, such that

$$(P+Q)\,EF = R \times CF.$$

Similarly, the resultant of $(P+Q+R)$ acting at F, and the force S at D is the force $(P+Q+R+S)$ acting at a point G such that

$$(P+Q+R)\,FG = S \times GD.$$

Proceeding in the same way we may find the resultant of any number of parallel forces acting at different points of a body.

If the forces P, Q, R, S etc., applied at the same points A, B, C, D etc., act in a different direction remaining still parallel to one another, their resultant will still pass through the same point G as before.

The point G is called the **centre** of the parallel forces. It is a point through which the resultant of any number of parallel forces passes, its position depending only on the magnitudes of the forces and the positions of the points at which they act, and is quite independent of their directions.

65. Centre of Gravity.—We have already seen that the weight of a body is the force with which the body is attracted by the earth towards its centre, the direction of action of this force being called the *vertical* direction at the place where the body is considerd.

Now we may consider a body to be made up of a large number of small particles rigidly connected together, and that each of these particles is acted on by its weight, proportional to its mass. These forces all act towards the earth's centre, but having regard to the long distance, nearly 4000 miles, of the body from the earth's centre, and the size of the body being small compared to that of the earth, the forces may be considered parallel. The weight of a body, as a whole, is then really the resultant of a system of parallel forces made up of the weights of all the particles which build up the body, acting at points where these particles are situated. The point at which this resultant acts is called the **centre of gravity** of the body.

In art. 64 it is seen that the position of the centre of a system of parallel forces is independent of the

direction of the forces. Although the direction of gravity can not be changed, for it is always vertical, it will amount to the same thing, if the body be simply rotated through any angle. The forces of the system remain to be of the same magnitude and act at the same points within the body, but the direction of the system has changed relative to any line in the body. It follows, therefore, that the centre of gravity of a body is *fixed relative to the body.*

The centre of gravity of a body is *not necessarily in the body itself*; it may be at a point outside the substance of the body. Thus the centre of gravity of a circular ring of wire is at the centre of the ring, that of an empty beaker is within the air enclosed by it.

It is not necessary again that a body should be rigid in order that it may have a centre of gravity. Thus we speak of the centre of gravity of a fluid mass, or the centre of gravity of a system of bodies not materially connected in any way.

The only condition for the centre of gravity of a body to be a fixed point relative to itself is that its size and shape should remain unaltered. If the body is made up of movable parts, the centre of gravity is fixed for any given configuration of the body, but changes its position with change of configuration. For example, a straight piece of uniform wire has its centre of gravity at its middle point, and its weight will act at that point as long as it remains straight. If the wire be bent, it will no longer have the same centre of gravity. In a draw-telescope the position of the centre of gravity varies as the tubes are more or less drawn out. Again, if a man raises his arm, his C. G. is displaced.

The centre of gravity of a body may, therefore, be defined as the point, fixed relative to the body, through

which the resultant of the weights of the paricles which build up the body, passes for all positions of the body, so long as its size and shape remain constant.

For all practical purposes, the centre of gravity is the point at which the whole weight of the body may be supposed to act.

The centre of gravity is often abbreviated as C. G.

66. Centre of Mass or Centroid.—If instead of considering the weight of a body we look to its mass only, and suppose the body to be acted upon by a system of parallel forces, such that the forces to which the individual particles of the body are subjected are all parallel in direction and proportional to the masses of the particles in magnitude then the centre of the system of parallel forces, through which their resultant may be supposed to act, is called the **centre of mass or centroid.** It may be said to be the point about which the mass of a body is evenly distributed. Every body, since it has a mass, has always a centre of mass. It is abbreviated as C. M.

We observe that the centre of gravity is a particular case only of the mass-centre, in which the forces are the vertical forces due to gravity and the C. G. of a body does *necessarily* coincide with its C. M.

In Mechanics, when a body is moving in any way, the velocity of its centre of mass is taken for the velocity of the body.

67. C. G. of Symmetrical Bodies.—In finding by calculation the centre of gravity of a body the theory of finding the resultant of a number of parallel forces acting at known points is to be applied in a manner suitable to the particular form of the body. From considerations of symmetry, however, we can at once note

the following results. It is assumed that the bodies herein considered are of uniform density throughout.

The centre of gravity of

(1) a straight piece of a uniform wire, stick, rod or beam etc., is the middle point of its axis ;

(2) a uniform circular lamina* or a sphere is its geometrical centre ;

(3) a uniform circular ring is the centre of the circle ;

(4) a uniform parallelogram lamina or a rectangular parallelopiped is at intersection of its diagonals ;

(5) a uniform triangular lamina is at the intersection of its medians.

68. Experimental Determination of the centre of gravity.—The centre of gravity of a lamina may also be found by experiment. It will be shown in art. 70 that when a body is suspended freely from a point and is in equilibrium, its C. G. is in the vertical line passing through the point of suspension.

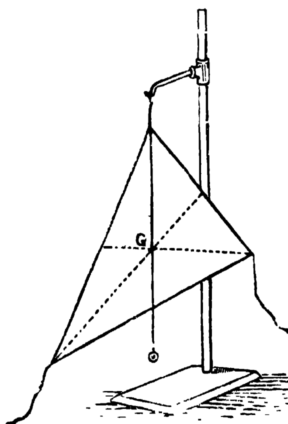


Fig. 56
Determination of C. G.
by suspension

Expt. 5. Suspend the lamina by a string attached to one corner of it. Trace on it the vertical line through the point of suspension by means of a plumb-line. The C. G. must be somewhere on this line.

*A *lamina* is a sheet of a material of small thickness, such as a sheet of paper, a thin sheet of metal etc. A *uniform* lamina is of the same thickness and is formed of the the same substance throughout.

Hang the lamina from another point, and draw the vertical line through the point of suspension as before. The C. G. must also be on this line. Hence, G, the point of intersection of these lines will be the centre of gravity required. To verify this, suspend the lamina from a third point, the vertical through which will also be found to pass through G.

The C. G. of a lamina or that of a card-board or a sheet of metal plate may also be found by balancing the body in two different positions on a horizontal edge. When the body is just balanced, its C. G. must be supported by the latter and hence must be vertically over some point at the edge.

Expt. 6. Balance a card-board against the edge of a table. Holding the card (Fig. 57) in this position, draw by a pencil a line on the nudersurface of the card, using the edge as a ruler. Turn the card to some other position and repeat the process. The point of intersection of the two lines thus marked indicates the position of the C. G. wanted.

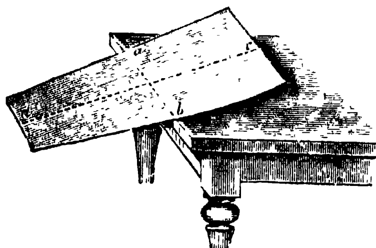


Fig 57

Determination of C. G. by balancing

Support the body on a pin-head placed at the C. G. thus found. The plate thus supported ought to be in equilibrium.

69. Equilibrium of Heavy Bodies—A body at rest under the action of forces which balance each other is said to be in **equilibrium**. As the weight of a body may be supposed to act vertically downwards through its centre of gravity, the condition of equilibrium in all cases is that the resultant of the reactions at the points of support to the body must act

vertically upwards and pass through the C. G. of the body. We notice the following cases:—

When a body is supported at one point, for example when it is suspended by a thread attached to a point (Fig. 58 a), or when it is balanced on a pivot (Fig. 58 b), or when it rests on a plane touching it at one point (Fig. 58 c), equilibrium is only possible when its C. G. either coincides with this point or is exactly

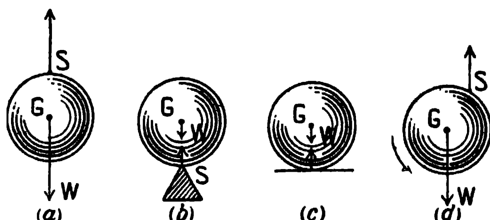


Fig. 58
A body supported at one point

above or below it in the same vertical line. For when a body is supported in this way the only forces acting on it are its own weight W acting vertically through G, its C. G., and the force supporting it or the force of reaction at the point of support S. If these two forces are to be in equilibrium they must be equal and opposite and in the same straight line. If it is not so, the weight of the body W, acting at G (Fig. 58 d) would cause a rotation about S until G and S are brought in the same vertical line.

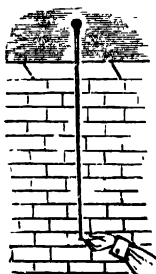
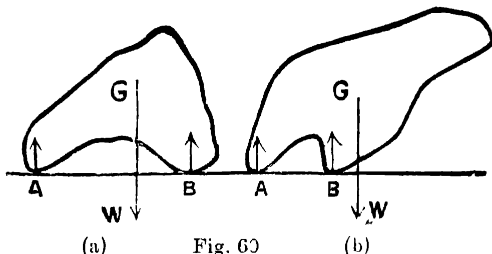


Fig. 59.
Balancing a stick.

Expt. 7. Try to balance a long stick on a finger-end. Note that unless care is always taken to keep the point of support vertically below the centre of gravity by the quick adjustment of the hand, the stick will fall. Also note that it is easier to

balance a long stick than a short one, for the C. G. of the former in falling through a greater height allows the finger more time to adjust the point of support.

If a body is supported on two points A and B on a table, the reactions on the table at A and B are both upward vertical forces, the resultant of which must



(a) Fig. 60 (b)

pass through some point between A and B in the line AB. Therefore the condition of equilibrium is that the vertical through the C. G. must fall between the two points of support on the line joining them. If the C. G. is in a position as shown in (Fig. 60 b), no equilibrium is possible ; if it has a position as in (Fig. 60 a) it can remain in equlibrium. Balancing on *stills* is an example of this case of equilibrium.

Lastly, consider the case of a body that rests on a plain surface on three points, for example, a three-legged table or on points more than three, for example, a glass tumbler resting on a table. Imagine a fine thread drawn *tightly* round the body so as to include all the points of contact with the supporting surface. The area thus enclosed is called the *base* of the body. Now the reaction of the table at the varius points of support all act verticalty upwards ; their resultant therefore acts vertically upwards at some point within the area of the base and this, since it balances the weight of the body, must pass though its C. G. Thus equilibrium is possible only when the

vertical through the C. G. falls within the area of the base (Fig. 61).

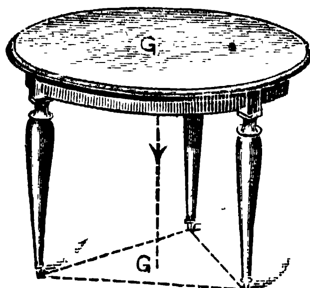


Fig. 61
Equilibrium of a table

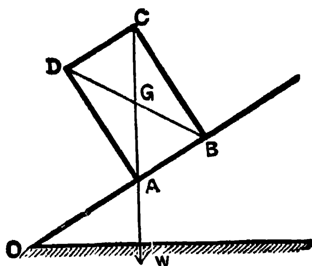


Fig. 62
A body about to topple over

The condition is the same whether a body rests on a horizontal surface or on an inclined plane.

The truth of the above fact can be tested by some simple experiments:—

Expt. 8. Take a piece of brick and place it on a *rough* plank and tilt the plank about an edge O. The body will not overturn so long as the vertical line through its C. G. falls within the base on which it rests (Fig. 62)

Now the C. G. of the body is evidently in the plane of the diagonal AC. Hence the brick will topple over as soon as the diagonal AC will pass through the vertical position.

Many illustrations to this point can be cited. If a cart is loaded with a very high load, so that C. G. of the cart and load together is high above the ground, it may be overturned by a small tilt caused by one wheel passing over a stone or a bank of earth, when the vertical through the C. G. falls outside the wheel

base (Fig. 63). For the same reason, a boat is liable to be upset easily, when the persons seated in it stand up, or when it is loaded to a great height.

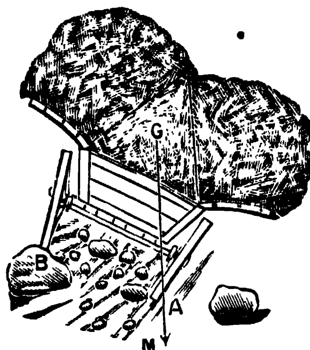


Fig. 63
A loaded cart about to overturn

In the ordinary upright position of a man the C.G. of his body is at about the middle of the lower half of his pelvis. But the C. G. is displaced when he carries a load. In order to retain the stability he must modify his attitude so as to bring back his C. G. over a point between his two feet. Thus a porter carrying a heavy trunk in one hand has to lean his body on the opposite side and often extends the other arm at full length. A man with a load on his back is obliged to lean forward. A person with an overgrowth in his abdominal front has to throw back his head and shoulders. In rope-dancing, the performer holds in his hand a long pole or an open umbrella to help him in maintaining the combined C. G. vertically above the rope.

Expt. 9. Stand sideways against a wall with a foot and the head both touching the wall and try to stand on that foot. It will seem to be impossible, because the wall will not allow the

C. G. of the body to be brought over the foot next to the wall.

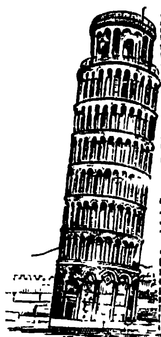


Fig. 64
The Leaning
Tower of Pisa

It follows that the wider the base on which a body rests, the greater is its stability, for then even with a considerable inclination the vertical through its C. G. still falls within its base. The well-known *Leaning Tower of Pisa*, (Fig. 64) from which were performed some of Galileo's famous experiments on falling bodies, is an illustration to the point. It is so much out of the vertical, that it seems ready to fall at any moment; yet it has remained in its present position for centuries.

70.—States of Equilibrium.—Although we have seen that a body is in equilibrium when the resultant of the supporting surface acts in the same vertical line passing through the C. G. of the body, it is yet possible to distinguish between the states of equilibrium. Equilibrium may be of three kinds, stable, unstable and neutral.

A body is said to be in **stable** equilibrium when it tends to come to its original equilibrium position after being *slightly* displaced. Thus for example, the plummet in fig. 58 (a) is in stable equilibrium, for if pulled aside and then released, it will at first swing to-and-fro, but will come at last to rest in the same position.

A body is said to be in an **unstable** equilibrium, when a body at rest, after receiving a small displacement, tends to move farther away from its equilibrium position. Thus a stick balanced on the

finger end (Fig. 59) or an egg on its end are in the state of unstable equilibrium.

Lastly, a body is said to be in a **neutral** equilibrium, when after being displaced, it neither returns to its original position nor moves farther from the new one. A uniform sphere fig. 58 (c) or a cylinder resting on a flat surface are in the state of neutral equilibrium.

A cone or an ordinary glass funnel affords a good illustration of all the three kinds of equilibrium. When it rests on its base, it is in a stable equilibrium (Fig. 65, A, A′) ; when balanced on its apex, it is in an unstable equilibrium (Fig. 65, B, B′) and while resting on its side, it is in the state of neutral equilibrium (Fig.65 C, C).

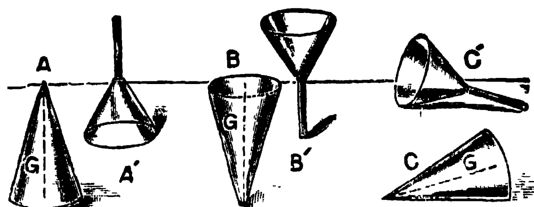


Fig 65
The three states of equilibrium

The three states of equilibrium of a body are, in fact, determined by the position of the centre of gravity of the body. In all cases of stable equilibrium the C. G. is as low as possible. The slightest displacement of the body elevates its centre of gravity.

Hence the body returns to its original position as soon as it is permitted to do so. When the equilibrium is unstable, the centre of gravity is as high as possible. Any slight displacement of the body will help its C. G. in coming down to a lower height. From the above it is seen that *the C. G. of a body*

tends to occupy the lowest possible position. In the neutral equilibrium of a body, its centre of gravity is neither raised nor lowered by any displacement given to the body.

When a body can rest on a plane on different bases, for example a book, the limits of stability widen for a position which allows the centre of gravity to be lowered. Thus a copy of the Encyclopædia Britannica has a greater stability when it lies flat on a table than when it stands on an edge. On the other hand, the stability of a body ordinarily in an unstable equilibrium may be increased by the addition of weights, so that the C. G. is brought under the point of support (Fig. 66).

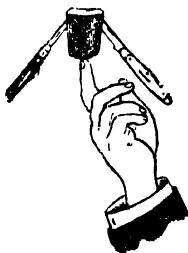


Fig. 66

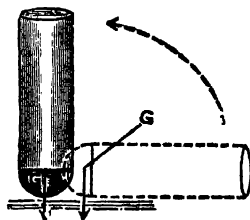


Fig. 67

The C. G. of a loaded cork

Expt. 10.—Fix two knives into a common cork on opposite sides (fig. 66). The cork will now very easily balance on the point of a finger or a pencil, and will not fall even when the point of support is shifted to one side of the base of the cork.

Some apparently paradoxical but illustrative and interesting experiments may be arranged on the point :—

Expt. 11.—Fix a cylinder of cork to a solid hemisphere of lead (fig. 67) by means of sealing-wax. As lead is much heavier than cork, the C. G of the whole is below the centre of the hemisphere. It will stand upright though it looks topheavy. Tilt it to one side to make it horizontal. The point G, its C. G.,

is thereby raised ; the weight of the body acting there makes it spring back to its original position as soon as the body is free.

Fig. 68 represents a similar toy called the *Tumbler* which consists of a light figure attached to a hemisphere of lead. When the figure is upright, its C. G. occupies the lowest position.

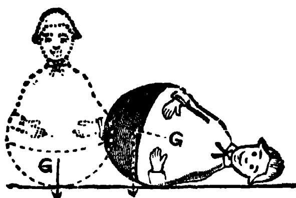


Fig. 68.
The tumbler

Fig. 69 affords another illustration. It is a disc of wood with a small mass of lead (shown shaded in the diagram) inserted within it near the edge. The point G is the combined centre of gravity.

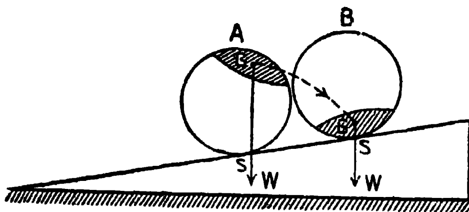


Fig. 69
A loaded disc on a tilted plane

Expt. 12.—Place the disc on a slightly inclined plane in the position A in which the vertical through G does not pass through the point of contact, S. The disc will ascend the plane, because thereby the point G comes to occupy a really lower

position with respect to the ground. After gliding up the plane to a certain extent, it stops and then moves down and settles to a state when the vertical through G passes through the point of contact.

Exercise VI

1. Define Centre of Gravity. How can you find practically the C. G. of an irregular plane ?

2. State in a general way when a body will stand or fall. Cite practical illustrations.

3. Define the three states of equilibrium. How does the position of the C. G. affect the equilibrium of the body ?

4. A circular table weighs 20 lbs. and rests on three legs in its circumference forming an equilateral triangle. Find the least pressure that must be applied at its edge to overturn it.

5. A telescope consists of three tubes each 10 in. in length sliding within one another, and their weights are 8, 7, 6, ozs. respectively. Find the position of the centre of gravity when the tubes are drawn out to their full lengths.— (*Lond. Matric.*)

6. A cylinder, whose base is a circle 1 ft. in diameter, and whose height is 3 ft., rests on a horizontal plane with its axis vertical. Find how high one edge of the base can be raised without causing the cylinder to turn over.

7. Weights of 1 lb., 2 lbs., 3 lbs. and 4 lbs., are suspended from a uniform lever 5 feet long at distances of 1 foot, 2 feet, 3 feet and 4 feet respectively from one end. If the mass of the lever is 4 lbs., find the position of the point about which it will balance.

8. A heavy beam consists of two portions, whose lengths are as 3 to 5, and whose weights are as 3 to 1 ; find the position of the centre of gravity.

9. A uniform plate of metal 10 inches square, has a hole of area 3 square inches cut out of it, the centre of the hole being $2\frac{1}{3}$ inches from the centre of the plate ; find the position of the centre of gravity of the remainder of the plate.

CHAPTER VIII

FRICTION

71. Resistance to Motion.—The forces which tend to oppose or destroy motion are, in general, called resistances. Thus a man dragging a heavy weight along the ground has to make a muscular effort to overcome the resistance of the ground opposing motion ; a body falling in air or a cyclist riding against the wind has to meet with the resistance of the air ; a ship in motion has to cut its way through water ; in a machine, a part of the work done by it is always spent in overcoming the frictional forces between the different parts in the machinery. **Friction**, in the widest sense of the term, may be used to mean any resistance to motion, but it is ordinarily used in a limited sense *viz., it is the resistance which a moving solid meets with on the surface of another solid which supports it.*

In the case of a body moving through a fluid medium, the moving body has to set in motion those parts of the medium with which it is in contact. The resistance encountered by the body is directed against its front side and increases with the velocity and the extent of exposed surface of the body. It also increases with the density of the medium. The resistance of the air serves to diminish the velocity of a rain-drop or a hail-stone which, falling from a height of a mile or so, would otherwise have attained the speed of a musket shot. Use is made, on the otherhand, of this resistance of air in a descent by parachute and in

regulating wind-vanes for diminishing the velocity of falling bodies.

72. Friction.—If two bodies be in contact with each other and a force be applied tending to make one body slide on the other, an opposing force is set up in the plane of contact of the surfaces in a direction tending to prevent the motion. This force is known as the force of friction between the surfaces in contact.

Fricton is due to the *roughness* of the surfaces in contact. If these surfaces be perfectly smooth, there would be no opposing force of friction. Practically, however, the surfaces of bodies are never perfectly smooth. The minute irregularities of one surface engage with the small inequalities of the other and thus always cause some force to act between the surfaces in contact, being directed so as to prevent any displacement of one surface relative to the other in the plane of contact. In other words, the resistance due to friction acts in a direction parallel to the surface.

Let a rectangular body B (Fig. 70) be placed with one of its plane faces resting on the plane, horizontal

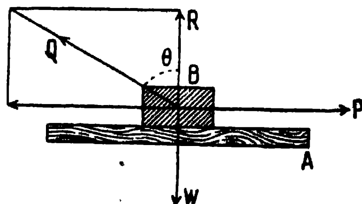


Fig. 70

Friction on a horizontal plane

surface of a table A. So long as B is at *rest*, the upward pressure R of the table balances the weight W of the block; these forces are both vertical

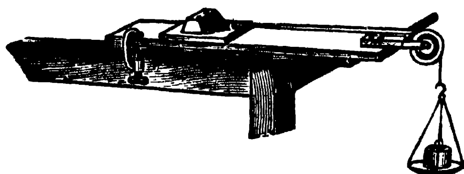
and there is no component in the direction of the surface, no friction being called into play. If a small force P be now applied to B parallel to the surface, a resistance F is felt which acting in the plane of contact prevents the block from sliding over the plate A. Sufficient friction is thus exerted just to stop the motion, F being exactly equal to P. If the forces F and R are compounded to give a single resultant Q making an angle θ with R, then this force Q is called the resultant reaction between the surfaces. As P is increased, the value of F also increases, being always equal to P and the angle θ also increases correspondingly. Friction is, therefore, *a self-adjusting force*; no more friction, however, is called into play than what is just sufficient to prevent motion. The friction exerted under these circumstances is called **static friction.**

But as P is increased indefinitely, the amount of friction F exerted at the plane of contact cannot evidently be unlimited. The force F soon reaches a certain maximum limit depending on the nature of the surfaces in contact and the pressure exerted between them. The maximum limit to the value of the force of friction exerted when one body is just on the point of sliding upon another body, is called the **limiting friction.** The angle θ has then its maximum value and is called the **limiting angle.**

When P is increased beyond the value of the limiting friction, motion of the block sets in. The friction exerted after motion is started is called **dynamic friction** and is found to be much less than the limiting friction.

Expt. 13. Clamp a large plank of wood on a table so as to be horizontal. Attach a light pulley to its end projecting off the table as in fig. 71. Place a rectangular block of wood on the plank to act as a sliding piece. The surfaces of the block and the plank of wood in contact should be made smooth and even by rubbing with sand-paper. The block is attached to a string which pass-

ing over the pulley is connected to a pan. The part of the string above the table should be horizontal.



Fig—71
Friction Apparatus

Begin the experiment by placing small weights on the pan and then go on adding until the block just begins to slip on the plank. Near about the slipping point the board should be gently tapped to get a fairly good result. Note the total weight on the pan ; this together with the weight of the pan itself measures the maximum friction exerted between the plank and the wood.

Repeat the experiment. It will be seen that the value obtained for the limiting friction is fairly constant.

It is to be understood, however, that the limiting value of the friction found in the last experiment is constant in the particular case considered. Any change of the conditions in the above experiments will cause the limiting friction to change in magnitude.

Expt. 14. Determine the maximum friction (*a*) when a wooden block is placed on the wooden plank having fibres at first parallel and then at right angels to each other ; (*b*) when the former is placed on a glass surface. It will be seen that the maximum friction in each case is different.

The results of these experiments show that the maximum friction depends on the nature of the surfaces in contact.

73. Laws of Friction.—The relation between the limiting friction between two surfaces in contact and the area of contact, as well as that between the limiting friction and the pressure exerted normally between the surfaces, may be determined by experimental facts.

Expt. 15. Set up the apparatus as in Expt. 13. Determine the maximum friction between the block and the plank when the block rests on the plane successively on each of its three of different areas. Thus the areas of the surfaces in contact have been altered but the normal pleasure between them, which equals the weight of the block, remains evidently the same.

It will be found that so long as the normal pressure between the surfaces is not changed, the limiting friction remains practically the same, and is independent of the area of the surface of contact.

Expt. 16. Determine the weight of the sliding piece A and let it be b. Find also p. the weight of pan. Set up the apparatus as in Expt. 13 and then determine the maximum friction using the unloaded block. The value of F is given by $p+w$, where w is the weight placed on the pan just sufficient to cause motion of the block.

Next place different loads on the block and determine the maximum friction in each case. The normal reaction R is given by $b+W$, where W is the weight of the load on the block. Tabulate the results.

The above experiment is performed without altering the area and the nature of the surface of contact. It will be found that the ratio of the limiting value of the friction to the normal pressure between the surfaces, *i.e.*, F/R, is constant.

We thus arrive at the following laws of friction :

(i) *Friction always opposes motion.*

(ii) *It depends on the nature of the surfaces in contact, but is independent of the extent of the areas in contact so long as the normal reaction between the two surfaces is the same.*

(iii) *The maximum friction between two surfaces is proportional to the total pressure between them.*

74. Coefficient of Friction.—The constant ratio of the limiting friction to the normal pressure for any two specified surfaces is called the coefficient of friction, and is generally denoted by μ. If F be

the limiting friction and R the normal force, we have,

$$F/R=\mu \quad \text{whence } F=\mu R \quad \ldots \quad (55)$$

The value of μ is obtained from the experimental determination of F as in Expt. 13. The determination may also be made by placing a block of any material on an ordinarily smooth horizontal surface which is to be gradually tilted until the block just begins to slide down. When this is the case, the friction, which acts here upwards along the plane to oppose the motion of the block *down* the plane, has reached its maximum value and just balances the component of the weight down the plane.

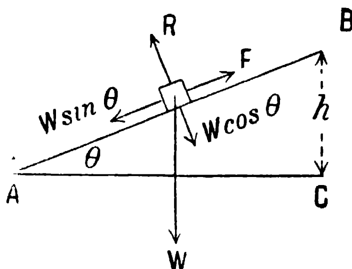


Fig. 72
Friction on a rough inclined plane.

Let θ be the inclination to the horizontal of the inclined plane AB of which the base is AC and the height BC. Let W be the weight of the block and R, the normal reaction (fig. 72). It follows from geometry that the angle which the normal to the plane makes with the vertical is equal to the angle of the plane. The components of W along and perpendicular to the plane are W sin θ and W cos θ respectively. When θ is of such a value that the block is just on the point of sliding, the component W sin θ is balanced by the maximum friction F and the normal reaction R also balances W cos θ. We have, therefore,

$$\mathrm{F}=\mathrm{W}\sin\theta$$

and $$\mathrm{R}=\mathrm{W}\cos\theta$$

$$\text{whence } F/R = \frac{W \sin\theta}{W \cos\theta} = \tan\theta$$

But $F/R = \mu$, the coefficient of friction

$$\therefore \quad \mu = \tan\theta \quad \dots \quad (56)$$

Tan θ again is obtained from the relation

$$\tan\theta = \frac{\text{height}}{\text{base}} \text{ of the plane} \quad \dots \quad (57)$$

This limiting angle θ is also called **the angle of repose.**

Thus the coefficient of friction is found by reading the height of the plane and dividing it by the base.

The approximate values of the coefficient of friction in a few common cases are given below :—

Surfaces in contact

Wood on wood fibres parallel	...	0·5
,, ,, fibres at right angles	...	0·33
Metal on wood	...	0·18
Metal on metal	...	0·6
Leather on meatal, dry ...	...	0·56
,, ,, oily ...	...	0·15

75. Dynamic Friction:—As stated before, limiting friction is the friction exerted between any two surfaces when motion is on the point of being started. As such, it is the maximum value of static of friction between the two surfaces and the coefficient of friction as determined above is sometimes called coefficient of static friction.

When the two surfaces are moving relatively to one another, the resistance of friction exerted between them is found by expteriments to be much less than when motion is on the point of starting, even though

the normal reaction is the same in both cases. The ratio of the friction to the normal reaction after motion is started is called coefficient of dynamic friction. It can be experimentally determined by the inclined plane method as described in the last article. The plane is continually tapped as its inclination is gradually increased until motion sets in. The inclination is then decreased until motion just stops. The coefficient of dynamic friction is the limiting value of tan θ just sufficient to keep up motion after it is started.

76. Rolling and Sliding Friction.—Friction is of two kinds, **sliding** and **rolling**. When a body slides over another, for example, when a heavy box is dragged along a floor, when the two hands are rubbed together, or when an axle of a wheel rotates, sliding continuously upon the same points of its bearings, the case is one of sliding friction. When a body, on the other hand, rolls over another as in the case of an ordinary wheel on a road, the case is one of rolling friction. Rolling friction is considerably less than the sliding friction, and there is a great saving of power when the latter is converted into the former. This explains the use of castors on heavy pieces of furnitures such as pianos, tables etc. Heavy weights like large blocks or stones are dragged along by supporting them on rollers (Fig. 73). In the ordinary wheels, the sliding friction is not, however, entirely removed, for the wheel slides continuously at some point A upon the axle (Fig. 75). In the *ball bearing* arrangement as in bicycles, the sliding friction is

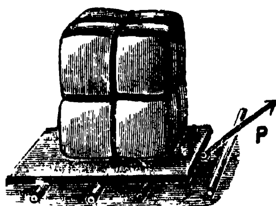


Fig. 73
Roller Bearing

completely replaced by rolling friction where a number of hard steel balls are loosely confined in a metal case round the axle ; the hub of the wheel rolls on them (Fig. 74).

To diminish friction various other methods are adopted. It is diminished by polishing the surfaces in contact. Lubricators such as oil, graphite, tallow are in frequent use to diminish the frictional resistance in machnies. In the motion of sledges over ice, the ice melts under pressure ; its surface acts as if lubricated and polished and hence the friction is much reduced. As a rule, greasy substances which are not absorbed by a body, diminish friction but

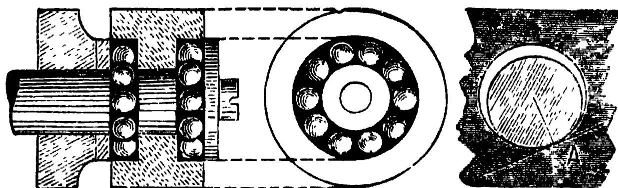


Fig. 74
Ball-Bearing

Fig. 75
Common Bearing

increase it if they are absorbed ; thus moisture and tallow increase the friction of wooden surfaces but diminish that between metal surfaces.

Although in some cases it is advantageous to reduce friction as far as possible, yet there are cases in which its presence is an indispensible necessity. Without friction of the ground one would not be able to walk ; no transmission of motion by belting, rope etc., would be possible. Without friction one could not climb on trees, tie knots or fix nails and textile fibres would fall to pieces.

Exercise VII

1. Explain the terms static friction, limiting friction and dynamic friction. What is meant by the angle of repose? How would you experimentally find a relation between the co-efficient of friction and the angle of repose?

2. What do you mean by *Friction?* Define co-efficient of friction.

3. The co-efficient of friction between a block and a table is 0·3. What force will be required to set the block weighing 400 gms. in motion?

4. A heavy body is just on the point of sliding on a rough plane that rises 3 in a length of 5; find the co-efficient of friction.

5. A body, of weight 6 lbs. rests in limiting equilibrium on a rough plane whose slope is 30°. The plane is next raised to a slope of 60°; find the force along the plane required to support the body.

6. Give instances of cases in which it is desired to have the friction increased, and others in which to have it diminished.

7. Describe a method of determining the co-efficient of friction between teak and iron.

8. An inclined plane is adjusted so that a flat-bottomed box placed on it just steadily moves down. What difference will you notice if a load of a kilogramme is placed on the box? Give reasons for your answer. [*Pat. U.—1919*]

CHAPTER IX

MACHINES

77. Machine.—A **machine** is a contrivance or an instrument by means of which a force, applied at a point and in a given direction, is able to exert itself at some other point, possibly in a different direction and with different intensity to overcome some other resisting force, *i.e.*, to do work.

The force impressed on the machine is called the **effort** or **power** and the resistance to be overcome is ordinarily called the **weight** or simply the **resistance**. The effort is generally denoted by P, and the resistance by either Q or W.

The term '*power*' is not a happy one to use in this sense, for it is now definitely used to mean *the rate of working* ; hence the word *effort* is preferred to it. Again, it is better to use the term *resistance* instead of *weight*, as machines are often used to overcome resistances other than those due to gravity.

78. Work done by a Machine.—It must be noticed at the outset that whether the force exerted by a machine is greater or not than the force impressed on it, the machine is unable to supply a greater quantity of energy than what is put into it ; in other words, no more work is done *by* it than is done *on* it. As a matter of fact, in every machine some amount of friction is bound to be present between its different parts. Hence a part of the energy supplied to a machine is lost in overcoming

this internal resistance within it, and the rest is the effective work done by it. Thus the larger the friction, the less is the energy utilized by the machine.

The ratio, which expresses what part of the total energy supplied to a machine is utilized by it, is called the **efficiency** of the machine. Thus

$$\text{Efficiency} = \frac{\text{Energy utilized}}{\text{Total energy supplied}}$$

The efficiency of a machine is always less than unity and is often multiplied by 100 and stated as a percentage. If we suppose a machine to be frictionless, or that the friction therein is negligible, the efficiency is unity and it follows from the fundamental principle known as the **principle of work** or the **conservation of energy** that *the work done by the effort is always equivalent to the work done against the resistance.*

Now work done by a force is measured by the product of two factors, *viz.*, the *force* and the *displacement* (see §. 96) of its point of application. It follows that if the machine be perfectly smooth throughout, and if P and W denote the power and resistance respectively, then

$$P \times \text{distance through which } P \text{ moves}$$
$$= W \times \text{distance through which } W \text{ moves.}$$

Thus if a small effort P overcomes a greater resistance W by means of a perfect machine, the point of application of P will have to move through a longer distance than that through which the point of application of W moves. This is popularly expressed as,—*what is gained in power, is lost in speed.*

79. Mechanical Advantage, Efficiency and Velocity Ratio of a Machine.—The ratio of the resistance to the effort which must be applied to a

machine to overcome it is called the **mechanical advantage** of the machine.

$$\text{Mechanical advantage} = \frac{\text{resistance}}{\text{effort}} = \frac{W}{P}$$

If the point of application of the resistance moves through a distance y when that of the effort moves through a distance x, then

$$\text{Efficiency} = \frac{\text{useful work done by the machine}}{\text{work expended upon it}} \quad (\S.\ 79)$$

$$= \frac{W.y}{P.x} = \text{mechanical advantage} \times \frac{y}{x}.$$

The ratio of the distance moved by the effort to that moved by the resistance is called the **velocity ratio**. Thus

$$\text{Velocity ratio} = \frac{x}{y}$$

and therefore $\text{Efficiency} = \dfrac{\text{mechanical advantage}}{\text{velocity ratio}}$

In a perfect machine of which the efficiency is unity,

Mechanical advantage = velocity ratio.

80. Simple Machines.—An ordinary machine *e.g.*, a pump, a steam-engine etc., consists of a number of simple parts which may be classified for separate study. Each of these parts is spoken of as a *simple machine.*

Simple machines are also called MECHANICAL POWERS.

The Simple Machines may be classified as :—

(i) The Lever, including the Wheel and Axle ,
(ii) The Pulley ;
(iii) The Inclined Plane, including the Wedge ;
(iv) The Screw.

81. The Lever.—The **lever** is a rigid bar, straight or bent, and is capable of turning about a fixed point of support. The fixed point is called the **fulcrum** and is denoted by F in the figures given below.

The perpendicular distances between the fulcrum and the lines of action of the effort and the resistance are called the *arms* of the lever. Thus in fig. 76 the arms are FL and FM.

The conditions of equilibrium in any case of lever are obtained from the Principle of Moments. The resultant of the forces P and Q impressed at A and B respectively, must pass through F. Hence the moment of P about F must be equal to that of Q about F. Thus

$$P \times FL = Q \times FM.$$

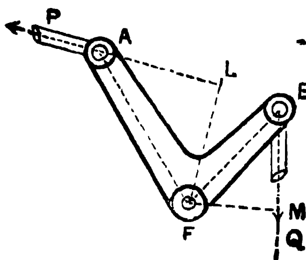


Fig. 76
A Bent Lever

The lever is most often a straight rod. We shall consider the cases when the lever is a straight one, and the effort P and the resistance Q are perpendicular to it. In theoretical calculations the thickness of the lever rod and its weight are neglected.

Levers of this kind are usually divided into three classes according to the position of the fulcrum with respect to the points of application of the effort and resistance.

82. The Three Classes of Levers ; Class I.—In this class the effort P and the resistance W acting on opposite sides of the fulcrum F keep the lever in equilibrium (Fig. 77).

Let R be the resultant of P and W, acting downwards through F. As the lever presses *downward*

on F with the force R, the reaction at F acts *upwards* with the same force R.

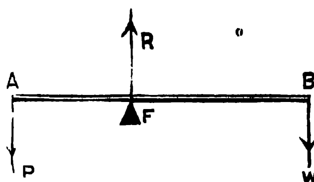


Fig. 77 Lever—Class I

As a condition of equilibrium, we have

$$R = P + W$$

as P and W are like parallel forces.

And the moment of P about F = the moment of W about F

i.e., $P \times FA = W \times FB$

The mechanical advantage is given by

$$\frac{W}{P} = \frac{FA}{FB} = \frac{a}{b} \qquad \ldots \qquad (58)$$

where a and b are the lengths of the arms FA and FB.

In this lever as a may be greater than, equal to or less than b according to the position of F along the rod, the mechanical advantage may also be greater than, equal to or less than unity.

The ordinary balance, in which the two arms are equal, is a special case of this class of lever.

Instances of the levers of this class :—A crow-bar, as ordinarily used to raise a weight, having its fulcrum at a point where it rests on a block near to the weight to be lifted (Fig. 78). A poker, used to raise coal in a grating ; a claw-hammer, when used

to extract nails ; a spade, in digging the earth ; a see-saw ; the handle of an ordinary pump ; an

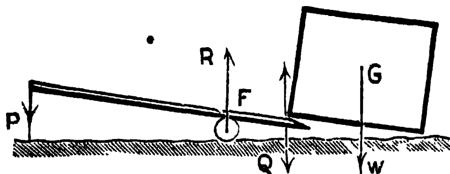


Fig. 78
A crow-bar used in raising a weight

ordinary balance ; the foot, when it is raised and the toe tapped on the ground, the ankle-joint being the fulcrum.

The oar of a rowing boat may be regarded as a lever of the first kind with the rowlock as the fulcrum, if the boat were kept at rest and the oar used to scoop the water backwards.

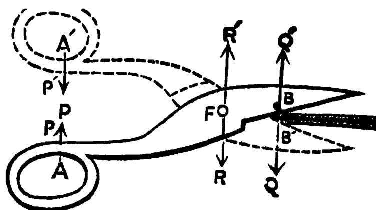


Fig. 79
A pair of scissors

pair of scissors (Fig 79), a pair of crucible tongs are instances of double levers of the First Class.

The principle of lever is said to be discovered by *Archimedes*, the noted geometrician of antiquity. It is related that at the launching of a huge ship designed by him, Archimedes displayed the power of a lever by using it for urging the ship off

the stocks, and that in reply to king Hiero's expression of wonder at the great force thus displayed, Archimedes uttered his famous boast, "Give me but a place to stand on, and I will raise the world."

Class II.—In the second class of levers, the effort P and the resistance W act on the same side of the fulcrum F, but in opposite directions, the effort acting at a greater distance from the fulcrum than the resistance (Fig. 80).

Here we have,

$$R = W - P$$

and $$P \times AF = W \times BF$$

Hence $$\frac{W}{P} = \frac{FA}{FB} = \frac{a}{b}$$

And since a is greater than b, the mechanical advantage is always greater than unity.

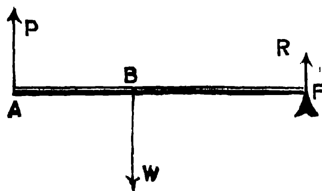


Fig. 80
Lever—Class II

Examples of this kind of lever : a wheel-barrow, in which the fulcrum is at the axle of the wheel and the power is applied at the handle : a crow-bar, when one end of it is in contact with the ground and power is applied at the other end : an oar of a boat, when the boat moves forwards, the end of the oar in water being the fulcrum which is, of course, not absolutely fixed : a cork-squeezer.

The raising of the body upon the toes in standing on tip-toe, or in the first stage of making a step

forward is an instance of the lever of the second class in the human body. Here the fulcrum is the ground on which the toes rest : the power is applied by the muscles of the calf to the heel : the resistance is the weight of the body borne by the ankle-joint.

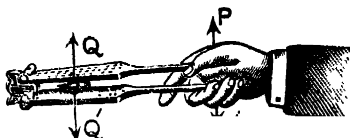


Fig. 81
A pair of nut-crackers

A pair of nut-crackers (Fig. 81) is a double lever of this class.

Class III.—The fulcrum in this case is at one end and the power P and the resistance W act on the same side of F as in class II, but the power acts nearer to the fulcrum than the weight (Fig. 82).

Here we have $R = P - W$

and $P \times AF = W \times BF$

therefore, $\frac{W}{P} = \frac{FA}{FB} = \frac{a}{b}$

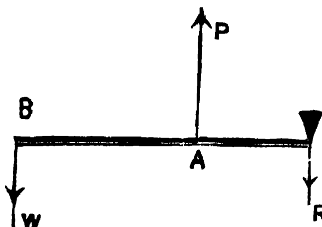


Fig. 82
Lever—Class III

and since here a is less than b, the mechanical advantage is less than unity. A small weight to be raised

requires a larger effort, but the point of application of the weight is considerably displaced when that of the effort moves through a small distance.

The treadle of a lathe or of a sewing machine is an example of this kind of lever. A very good example is seen in the bone of the fore-arm (Fig. 83), where the fulcrum is the elbow-joint; the effort is applied by the contraction of the biceps muscle, the lower end of which is attached to the fore-arm not far from the joint, and the weight is placed on the hand. Here rapidity of action is obtained at a loss of power.

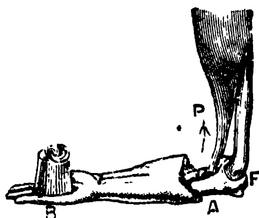


Fig. 83.
The fore-arm as a lever

A pair of sugar tongs, a pair of forceps in a weight-box are double levers of this class.

83. The Wheel and Axle.—The wheel and axle is a modification of the lever. It consists of two cylinders having a common axis (Fig. 84), the larger of which is called the *wheel* and the smaller the *axle*, the axis common to both being horizontal. The axis terminates in two pivots which can turn freely on fixed supports. Round the axle is coiled a rope, one end of which is fixed to the axle, while the other end supports the weight W. Round the wheel is coiled a second rope in direction opposite to the first, having one end attached to the wheel and

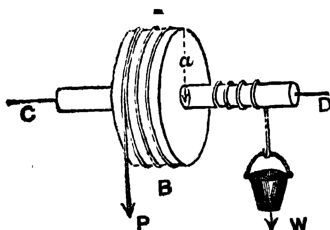


Fig. 84
Wheel and Axle

having the power P applied to the other end. Thus when P is lowered, the rope round the wheel unwinds, that round the axle coils up and raises the weight.

The condition of equilibrium is that the moments of the effort P and weight W about the axis must be equal and opposite,

i.e., $P.a = W.b$

where a and b are the radii of the wheel and axle respectively.

Hence the mechanical advantage is

$$\frac{W}{P} = \frac{a}{b} = \frac{\text{radius of wheel}}{\text{radius of axle}} \qquad (59)$$

By making the wheel larger and the axle smaller, the mechanical advantage may be increased : but in practice this has a limit, for the axle cannot be made too thin and thereby too weak, nor the wheel too large and cumbrous.

The above result can also be obtained from the Principle of Work. Let the wheel and axle be rotated through one complete turn. Then a length of rope equal to $2\pi a$ unwinds from off the wheel and a length equal to $2\pi b$ coils round the axle. Hence work done by P $= P \times 2\pi a$ and work done against W $= W \times 2\pi b$

$\therefore P \times 2\pi a = W \times 2\pi b$

whence $P.a = W.b$.

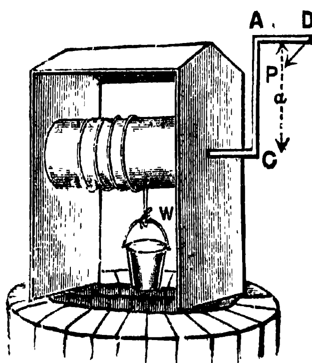


Fig. 85
A windlass

The windlass (Fig. 85) and the capstan (Fig. 86) are modifications of the wheel and axle. In

the **windlass** which is used for raising water from a well, the wheel is replaced by a crank-handle. In the **capstan** which is used on board a ship for raising the anchor, the axis is vertical. One or more men may apply force by pushing a number of horizontal projecting arms (called the hand-pikes). Here the moment of the pull along the rope is equivalent to the sum of the moments of the forces exerted by themen.

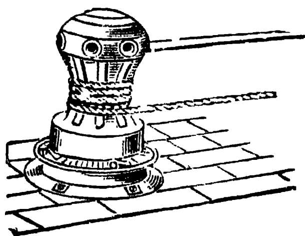


Fig. 86
A capstan.

The method generally employed to obtain rotatory motion by means of a belting passing round two wheels, or a linked chain passing over two toothed wheels (as in a bicycle) is another application of the wheel-and-axle arrangement. If the circumference of the large driving wheel is twice that of the smaller wheel, the latter

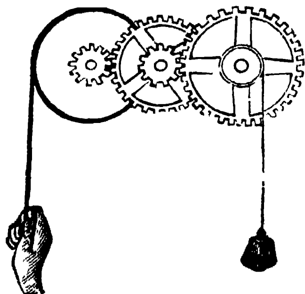


Fig. 87
Cog-wheels.

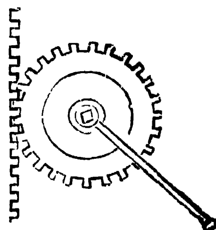


Fig. 88
Rack and Pinion.

will rotate twice as many times as the former in the same time.

A train of cog-wheels (Fig. 87) which is used in clocks, watches and speed-recorders is virtually a combination of wheels and axles. Every wheel with its axle or pinion on the same axis is a wheel-and-axle arrangement. The mechanical advantage of the whole system is the product of the mechanical advantage of each pair.

Fig. 88 represents a **rack and pinion** arrangement. It may be looked upon as a variety of the wheel-and-axle arrangement, in which the rack, a straight bar fitted with teeth, is to be regarded as a portion of a wheel of an infinitely large diameter. When the pinion wheel is rotated on a fixed shaft, its motion is converted into a straight one in the rack. The piston of a double-barrelled air-pump, the focussing arrangement of a telescope or of a microscope etc., are worked by such a contrivance.

84. The Pulley.—The **pulley** is a small circular disc or wheel of wood or metal with a groove cut round its rim to receive a string or cord which passes over it. The pulley can revolve freely about an axle, passing through its centre perpendicular to its plane, the ends of the axle being supported in a frame-work, called the *Block*.

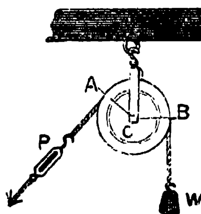


Fig. 88
A fixed pulley

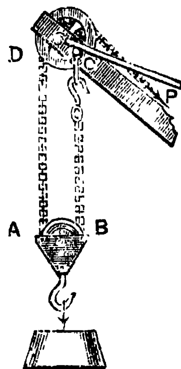


Fig. 89
A movable pulley

If the block be fixed as in fig. 89, the pulley is said to be *fixed*. When the block can ascend or descend as in fig. 90, the pulley is said to be *movable.*

For elementary study we suppose the pulley to be smooth, so that the tension of the string passing round it is the same throughout. Further, the weights of the pulley and the rope are often so small compared with the weights supported, that these may be regarded as negligible.

In the fixed pulley, the weight is attached to one end of a string passing over the groove, and the power is applied by pulling the other end. The fixed pulley is useful only in changing the direction of a force. Such a pulley is used for drawing curtains, hanging lamps, in pulling punkhas, raising weights etc.

Taking moments about the centre of the wheel, we have (Fig. 89),

$$P.CA = W.CB$$

But $$CA = CB$$

Hence $$P = W \qquad \ldots (60)$$

$$\therefore \text{ Mechanical advantage} = \frac{W}{P} = 1$$

In a *single movable pulley*, the weight is attached to a block, a string passing round the pulley is secured to a fixed support; the power is applied at the other end (Fig. 90).

When the strings are *parallel*, the tension along the two parallel strings supports the weight acting downwards. Moreover, if the pulley be assumed to be frictionless, the tension in any part of the string is the same, say P.

Hence $$2P = W$$

$$\text{Mechanical advantage} = \frac{W}{P} = 2 \qquad \ldots (61)$$

i. e., a given power can in this case raise twice its weight.

Pulleys are often combined in various ways in order to secure greater mechanical advantages. The most common arrangement known as the BLOCK AND TACKLE is employed on account of its superior portability (Fig.91). In this, the pulleys are arranged in

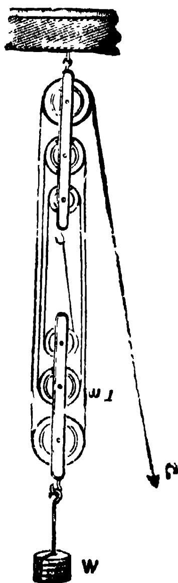


Fig. 91
Block and Tack'e

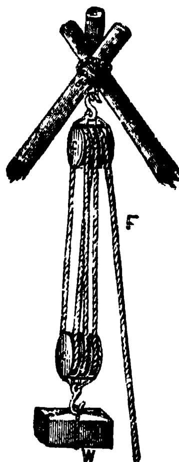


Fig. 92
Second system of pulleys.

two blocks, one fixed and the other movable and attached to the weight. The string which is continuous is attached to the movable block and passes alternately round the blocks, power being applied at the free end of the string.

The pulleys are sometimes arranged with a common axis; again, the various pulleys in a block are sometimes placed one below the other as in fig 92.

In either case, the tension of the string is equal to the power. Let n be the number of strings supporting the lower block. Then the total upward force at the lower block is nP. The downward force is W, the weight supported together with the weight of the lower block, ω say. Then $nP = W + \omega$.

If the weight of the lower block is neglected, then

$$nP = W \qquad \dots (62).$$

$$\therefore \text{ Mechanical advantage } \frac{W}{P} = n.$$

85. The Inclined Plane.—A plane, inclined to the horizontal plane at any angle α, is an **inclined plane.** By means of it a heavy body can be raised to a height by the application of a force less than the actual weight of the body, the friction on it being supposed to be negligibly small.

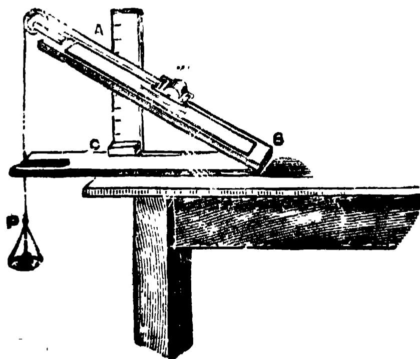


Fig. 93 (a)
An Inclined plane

In fig. 93 (a) ABC is an inclined plane with an inclination of α.

AB is called the *length* (*l*), *BC* the *base* (*b*), and *AC* the *height* (*h*) of the plane.

Let W be the weight of the body, and P the force continuously acting on it in a direction parallel to the plane.

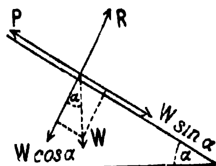


Fig. 93(b)

The component of W perpendicular to the plane
$= W \cos \alpha = R$, the normal reaction of the plane.

The component of W parallel to the plane, which only is effective in dragging the body down the inclined plane is

$= W \sin \alpha$. This must be balanced by P.

$$\therefore \quad P = W \sin \alpha \qquad \ldots (63)$$

$$\therefore \text{ Mechanical advantage} = \frac{W}{P} = \frac{1}{\sin \alpha} = \frac{l}{h} \qquad \ldots (64)$$

The above relation may also be obtained from the principle of work :

Work done by P in drawing the body up the whole length of the plane = P × *l*.

Work done by W against gravity

= W × the vertical height through which W is raised

$= W \times h$.

By the principle of work,

$$P \times l = W \times h$$

$$\therefore \text{ Mechanical advantage} = \frac{W}{P} = \frac{l}{h}$$

The inclination of an inclined plane may be indicated in either of the following ways:

(i) by the angle of inclination, α.

(ii) by any two of the three parameters l, b and h.

(iii) by a ratio y/x, where y denotes the vertical height of the plane corresponding to a length of base x.

(iv) by a ratio y/l, where y denotes the vertical height of the plane corresponding to a length l along the plane.

The ratios are expressed as y in l, y vertical to l along the slope, and so on. A plane having an inclination of 1 in 100 is one whose height is 1 unit of length for a length of 100 units along the plane.

If a railway engine has to pull a train up a hill with an inclination of 1 in 50, it has to exert a force equal to $\frac{1}{50}$ only of the weight of the train in addition to what it will have to exert on a level railway merely to overcome the friction. The principle is applied in practice in loading or unloading heavy goods into or out of a wagon by means of two inclined beams connected by iron ties ; in following a zigzag course in climbing a hill or going up a long stair-case etc. Again, the smaller the slope, the easier is the ascent ; this is also seen in the case of common ladder or a staircase in a house.

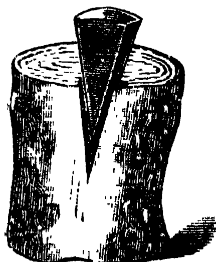


Fig. 94
Wedge.

The **wedge** is a double inclined plane, movable instead of being fixed as in the case considered, made of iron or some hard material and used in splitting wood (Fig. 94), in lifting a weight such as raising large blocks in order to put rollers or

chains under them. Knives, chisels, axes, choppers and many other cutting instruments are thin wedges with sharp edges.

86. The Screw.—Every one is familiar with a screw. It is essentially an inclined plane wound on a cylinder.

Expt. 17. Take a piece of paper ABC cut into the shape of a right-angled inclind plane of a small angle. Colour the edge AB and wrap the paper round a cylinder, say a common pencil (Fig 95), so that the base is at right-angles to the axis of the cylinder. The edge AB will form a spiral curve on the cylinder and will trace a screw.

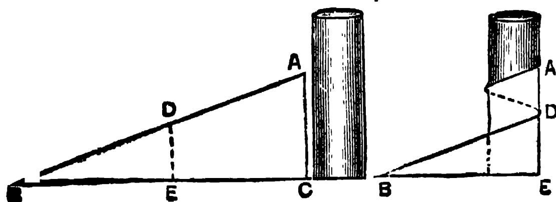


Fig. 95
The screw is essentially an inclined plane

Thus a screw consists of a cylinder of metal whose surface carries a uniform projecting thread, or has a groove cut on it along a spiral curve, making a constant angle to lines parallel to the axis of the cylinder. The section of the thread of the screw may be of different shapes *e.g.*, square, V-shaped etc.

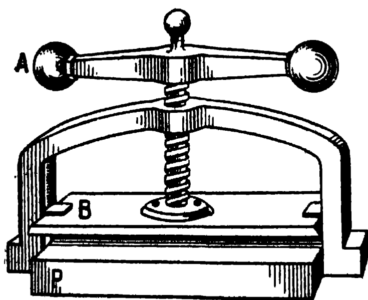


Fig. 96
Screw-press

The angle which the screw-thread makes with a

plane at right angles to the axis is called the ANGLE of the screw. This is the angle ABC of the inclined plane which traces the screw.

The distance between two consecutive threads, measured parallel to the axis such as AD, is called the **pitch** of the screw (cf. §15). If there be 15 threads for each inch of the axis of the cylinder, the pitch of the screw is said to be 1/15 inch or 15 threads to an inch.

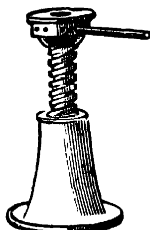


Fig. 97
Jack-screw

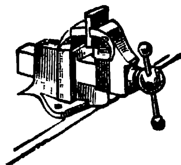


Fig. 98
Vice.

The screw is extensively used for fixing two bodies together. As a contrivance for exerting a great pressure it has its practical application in a vice, printing-press, oil-press etc. The jack-screw is used for lifting weights. Screws are used in the laboratory to produce small motion, *e.g.*, the levelling screws. Very small lengths are again measured by a micro-metre screw, *e.g.*, a screw-gauge, a spherometer, etc.

We shall consider the application of the screw in the **screw press** shown in fig. 96. It consists of a screw provided with a cross arm A at one end and a rectangular metal plate B at the other. The screw works in a frame work attached to a metal platform P over which the articles to be pressed are placed. When the cross arm is rotated in the right way, the plate B is lowered until presses against the platform. In each complete revolution of the cross arm, the plate is lowered through a vertical distance d which is equal to the pitch of the screw. Let r be

the radius of the cross arm and let a force P act at each of its ends at right angles to it. In a complete revolution, the point of application of each force moves through a distance $2\pi r$ so that the total work done is $2 \times 2\pi r$ P or $4\pi r$ P.

If F is the force exerted by the plate B, the work done by it is Fd. Then, by the principle of work,

$$4\pi r \text{ P} = \text{F}d$$

$$Or \ \frac{\text{F}}{\text{P}} = \frac{4\pi r}{d}$$

Thus if r is greater than d, which is always the case, a small force applied at the arm appears as a much greater force exerted by the plate B.

1). The **end-less screw** (Fig. 99) is a combination of a screw with a wheel and axle. The screw-thread fits in a toothed wheel in such a way that when the screw rotates, the wheel moves forward one tooth for each turn. It is to be noted that here the screw does not advance. A rapid motion of the screw-shaft is converted into a slow motion of the wheel. The endless screw is employed in many instruments for registering speed.

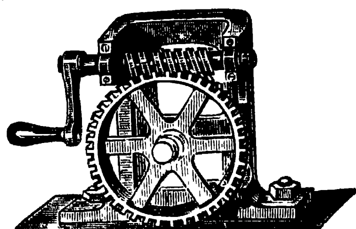


Fig. 99

End-less screw

87. The Balance.—The Balance is used for comparing the masses of two bodies, or rather for determining the mass of a body in terms of a standard mass. This is, of course, done by comparing the weights of the bodies, as weights are proportional to their masses at the same place.

A description of the common balance has already

been given in art. 17. It consists of a lever of the first kind with its fulcrum in the middle and placed a little above its centre of gravity. From the ends of the arms of the lever two equal and similar scale pans are suspended ; the mass to be weighed is placed on one of these and is balanced by placing suitable weights on the other. The arms of a balance ought, as we shall see latter on, to be equal and similar, if the balance is to be accurate.

In a good balance, steel knife-edges are used to diminish friction ; one at the fulcrum and two at the ends of the two arms. The fulcrum consists of a wedge-shaped piece of hard steel, whose fine edge is horizontal and perpendicular to the length of the beam and rests on hard plates of steel or agate. The scale-pans are attached to plates of steel or agate, which rest on similar knife-edges, fixed at the extremeties of the beam with their edges turned upwards. A needle or a pointer is fixed to the beam near the fulcrum ; the lower end of the pointer moves over a horizontal scale, such that when the beam is horizontal, the pointer is vertical and points to the zero graduation of the scale.

88. Requisites of a Good Balance.—A good balance should be so constructed that it is (1) *true*, (2) *sensitive and* (3) *stable.*

(1) A balance is said to be **true** if the beam be horizontal whenever bodies of equal weights are placed on the scale-pans.

The conditions required so that a balance may be true are that—(i) *The centre of gravity of the beam must be vertically below the fulcrum when the beam is horizontal.*

For, the condition of stable equilibrium requires that G, the centre of gravity and F, the fulcrum must be on the same vertical line. Then if G be above F,

the beam would be unstable, if G coincides with F, the beam would not oscillate and would, if slightly disturbed, continue to retain its new position but if G be below the fulcrum, the weight of the beam in its inclined position continually tends to bring it back to the original horizontal position ; in other words, the balance oscillates with regularity.

(ii) *the two arms of the balance must be equal.* In fig. 100, let

W = weight of the beam acting through G, its centre of gravity,

S,S′ = weights of the scale-pans respectively, and $AF = a$ and $BF = b$.

Now suppose the pans are empty and the beam is horizontal. The only forces which have a moment

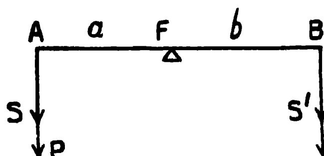


Fig. 100

about F are the weights S′ and S of the pans acting vertically through A and B respectively. Taking moments about F, we have

$$S.a = S'b \quad \ldots \quad \text{(i)}$$

Let two equal masses P and P be now placed on the two scale-pans : if the balance be true, the beam will still be horizontal. We have then

$$(P+S).\,a = (P+S').\,b$$

and from eqr.(i), $P.\,a = P.\,b$

i.e., $a = b$

Hence the arms should be equal

(iii) *the Scale- pans must be of equal weights.*

For, unless this is so, the beam cannot be horizontal when the arms are equal (eqn. i).

(2) A balance is said to be **sensitive** when the beam deviates appreciably from its horizontal position for a very small difference between the weights on the scale-pans. A good chemical balance will indicate a difference of weight down to a tenth of a milligramme.

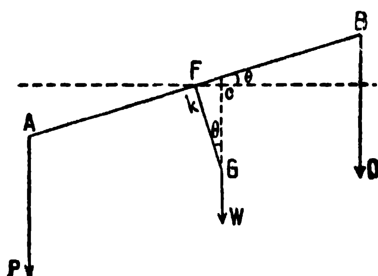


Fig. 101

Let two unequal masses P and Q be placed on the two scale-pans and let the beam be thereby inclined by an angle θ to the horizontal position. For a given value of $P-Q$, the greater the inclination θ, the more sensitive is the balance ; also the less the difference of P and Q, required to produce an inclination θ, the greater is the sensitiveness of the balance.

Hence the sensitivity of a balance $\propto \dfrac{\theta}{P-Q}$

The conditions that a balance may be sensitive are that

(i) *the centre of gravity of the beam shall be very near the fulcrum* ;

(ii) *the beam should be light ;*

To find the conditions of sensitiveness we assume that the balance is true. Let the centre of gravity of the beam be at G (Fig. 101), a distance k below the fulcrum. When the beam is inclined, its weight W

will have a moment equal to W.k sin θ tending to restore it. In order that a balance may be sensitive, *i.e.*, the inclination for a small difference of P and Q may be considerable, this moment of W about F must be small. This can be secured by making either k or W or both small.

(iii) *the arms should be long*

This condition is obtained from the fact that the inclination of the beam will also be great, if the moment of the difference of the weights on the pans is large. This moment, since the balance is true, is $(P-Q)a$. Thus the sensitiveness of a balance may be increased by increasing a, the length of the arms.

(3) A balance is said to be **stable**, when the beam after being disturbed, quickly resumes its original position of equilibrium. A balance would evidently be useless for weighing, if its equilibrium were *unstable* or even *neutral*.

For this, it is necessary that, when the scale-pans are equally loaded, the beam after displacement should come back rapidly to its position of equilibrium. Now in the inclined position of the beam the only moment that tends to restore it to its original position is that of the weight of the beam.

For great stability and for a given weight W of the beam, therefore, k must be large; in other words, *the centre of gravity must be well below the fulcrum.*

It will be noticed that a balance is most sensitive when k is very small, and most stable when k is large; thus the conditions of *sensitiveness* and *quick weighing* are to a certain extent contradictory. In practice, this does not affect much, since the purpose, for which a balance is required, determines the relative importance between the two conditions.

Thus for a balance used for ordinary commercial purposes such as in weighing coal, the main points are stability and rapid action ; for a balance used for research work in a laboratory, quickness of weighing may be sacrificed for sensitiveness.

For an ordinary good balance in a laboratory, a fair sensitiveness and reasonable rapid action can be secured by making k not very small and allowing the balance to have light and long arms.

89. Double weighing.—In any case where the accuracy of a balance is doubted, either of the two following methods of *double weighing* is adopted to find the true weight of a body :—

(*a*) BORDA'S METHOD OF SUBSTITUTION.—The body to be weighed is placed on the right-hand pan and is counterpoised exactly with fine sand or small shots placed in the opposite pan. Then the body is removed and replaced by standard weights from a weight-box until an exact balance is obtained. These weights are evidently equal to the required weight of the body, whether the balance is false or true, for these as well as the body are placed on the same pan and produce the same effect under exactly the same circumstances.

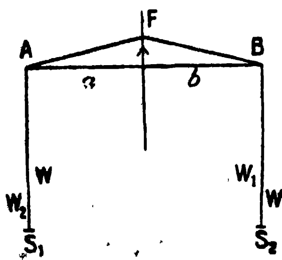


Fig. 102
Double weighing

(*b*) GAUSS' METHOD.—The body is weighed in the ordinary way first in one pan and then in the other. If the two observed weights are equal, each is equal to the weight of the body and the balance is true ; if not, the balance is false.

Let us assume that the balance is in exact equili-

brium when there is no load on the pans, and that it is false due to unequal lengths of the arms.

Let a and b be the lenghs of the arms and let a body whose true weight is W, appear to weigh W_1 and W_2 when placed on pans S and S_2 respectively.

We have

$$W.\ a = W_1.\ b$$

and $$W_2.\ a = W.b$$

By cross-multiplication,

$$W^2.\ ab = W_1.\ W_2.\ ab$$

or $$W = \sqrt{W_1.\ W_2} \quad \ldots \quad \ldots \quad (65)$$

i.e., the true weight W is the greometric mean of the observed weights W_1 and W_2.

Gauss' method is superior to Borda's, for it is quicker and the two readings in it can act as a check to each other.

When, however, W_1 and W_2 are very nearly equal as they generally are, so that

$$W_1 = W_2 = \delta$$

where δ is small, a sufficiently accurate result is obtained by taking the arithmetic mean instead of the geometric mean of W_1 and W_2.

For $W = \sqrt{W_1 \times (W_1 + \delta)}$, from eqn. (65)

$$= W_1 \sqrt{1 + \frac{\delta}{W_1}}$$

$$= W_1\left(1 + \frac{\delta}{2W_1}\right.$$

Neglecting higher powers of δ,

$$= W_1 + \frac{\delta}{2}$$

$$= W_1 + \frac{W_2 - W_1}{2}$$

$$= \frac{W_1 + W_2}{}$$

The ratio of the lengths of the arms may also be obtained from the formulæ given above :

For $\frac{a}{b}=\frac{W_1}{W}$ and $\frac{a}{b}=\frac{W}{W_2}$.

$$\therefore \quad \frac{a}{b}=\sqrt{\frac{W_1 \times W}{W \times W_2}}=\sqrt{\frac{W_1}{W_2}} \quad \dots (66)$$

Again, suppose that a tradesman uses a false balance having unequal arms of lengths a and b, and weighs out to a customer a certain quantity of a commodity which balances a weight W placed on one scale-pan. He then removes the weight W to the other scale-pan and weighs a further quantity of the same commodity. He is under the impression that he gives a quantity 2W whereas really he gives a quantity W_1+W_2, where W_1 and W_2 are the true weights which balance the weight W when placed on the two scale-pans.

Now $W_1 . a = W . b$
and $W_2 . b = W . a$

$$\therefore \quad W_1+W_2-2W = W.\frac{b}{a}+W.\frac{a}{b}-2W$$

$$= W\left(\frac{a^2+b^2-2ab}{ab}\right)$$

$$= W\frac{(a-b)^2}{ab},$$

which is always positive so long as a and b are unequal. It follows that

$$W_1+W_2>2W$$

i.e., the tradesman is a loser and loses by

$$W.\frac{(a-b)^2}{ab}$$

90. The Platform Balance.—When very heavy and bulky loads are to be weighed the ordinary balance becomes useless. We then take recourse to

weighing machines, such as the **platform balance** or the weighbridge,

Fig. 103 represents a platform balance. The platform P rests on two knife-edges *a* and *b* which are near the fulcrums F_1 and F_2.

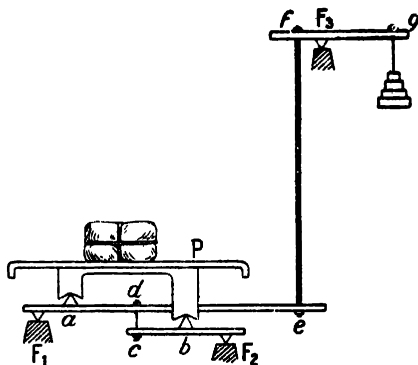


Fig. 103
A platform balance.

When a load is placed on P, the pressure is distributed at the points *a* and *b*. A short vertical rod at *c* connects the end of the lever cbF_2 to a point *d* on the lever $edaF$. Thus the pressure at *a* and *b* due to the load is communicated to the far end *e* of the lever $edaF_1$. Since F_2c is greater than bF_2 and eF is greater than both aF_1 and dF_1, it is evident that the downward pressure exerted at *e* due to the load is much smaller than the weight of the load. Again the end *e* of the lever $edaF_1$ is attached by a vertical rod to the end *f* of a third lever fF_3g close to its fulcrum F_3. Hence the far end *g* of this lever experiences an upward force which is proportional to but much smaller than the weight of the load

placed on P. This force is counterbalanced by weights which are so calibrated as to give the actual weight of the load. Fractions of the smallest weight suspended from g are obtained by sliding an equal weight along the arm gF_8 having calibrated graduations engraved on its edge.

The **weighbridge** is another form of a weighing machine used to weigh still heavier loads than can be weighed by means of the platform balance, such as a wagon of coal, a cart full of sugar-canes and so on. It also consists of a system of levers and works essentially on the same principle as that of the platform balance.

Exercise.—VIII

1. A wheel and axle is used to raise a bucket from a well. The radius of the wheel is 15 ins., and while it makes 7 revolutions, the bucket which weighs 80 lbs., rises $5\frac{1}{2}$ ft. Show what is the smallest force that can be employed to turn the wheel. [*Lond. Matric.*]

2. Find the inclination of a plane, if a horizontal force of 5 kilograms' weight can just move a mass of 12 kilogrammes.

3. A lever is 18 inches long. Where must the fulcrum be placed in order that a weight of 10 lbs., at one end may balance double its weight at the other end ?

4. A man whose weight is 200 lbs. is seated in a loop at one end of a rope passing over a smooth fixed pulley, and he holds the other end of the rope with both hands. Find the weight supported by each of his hands, supposing that they supported equal weights and that the two portions of the rope are parallel.

5. If there are six parts of the string at the lower block of a block and tackle, find the greatest weight which a man weighing 10 stones can possibly support.

6. A man raises a 4 ft. cube of stone, weighing 2 tons, by means of a crow-bar 8 ft. long, after having thrust one end of the bar under the stone to a distance of 6 inches ; what force must be applied at the other end of the bar to raise the stone ?

7. In a pair of nut-crackers, 6 inches long, if the nut be placed at a distance of 1 inch from the hinge, a force equal to $8\frac{1}{2}$ lbs. wt. applied to the ends of the arms will crack the nut. What weight placed on the top of the nut will crack it ?

8. Describe with a sketch the balance you have used in your Laboratory.

What are the requisites of a good balance ? [*C. U.—1922*].

9. Describe a *platform* balance.

CHAPTER X.

PENDULUM

91. The Pendulum.—If a body is suspended in such a way that it can oscillate freely about a fixed point or line as axis, it is called a **pendulum** or more correctly a **compound** or a **physical** pendulum. The pendulum of an ordinary clock is a particular type of the compound pendulum and is represented in fig. 104(*a*). It consists of a metallic rod A which turns about an axis at the upper end and carries at its lower end a heavy, lens-shaped mass of metal B called the **bob**, which can be raised or lowered along the rod by means of a screw S.



Fig. 104(a)
A compound pendulum

A **simple** pendulum consists of a heavy *particle* suspended by an inextensible string, *without weight*, and oscillating *without friction* about a point to which the upper end of the string is attached. Such a pendulum cannot evidently be realised in practice. A small sphere of lead or some other metal, suspended by a thread so fine that its mass and weight may be negligible, forms a close approximation to a simple pendulum.

Fig 104(b) represents a simple pendulum. A is the metal sphere called the bob, suspended by the string OP from O the **point of suspension**. The distance between the point of suspension and the centre of gravity of the sphere, *i.e.*, the geometrical centre, is called the length of the pendulum.

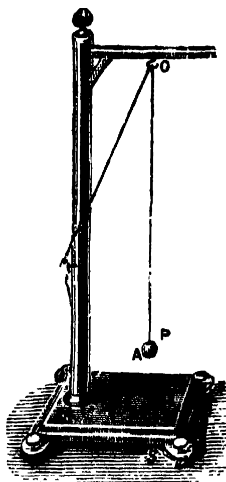


Fig. 104(b)
A simple pendulum

When the bob is drawn aside from its position of rest and then let off, the pendulum oscillates to and fro in a vertical plane. It is said to perform one oscillation or, more strictly, one complete oscillation when starting, from any point in its path in a given direction it comes back to the same point while moving in the same direction. Thus in fig. 105 when the bob starts from A, moves towards B, retraces its path to A, moves further to C and then comes back to A, it performs one complete oscillation.

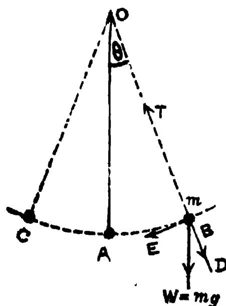


Fig. 105
Motion of a simple pendulum

The displacement of the bob from the position of rest to the farthest position on either side is called the amplitude and the time taken to perform one com-

plete oscillation is called the period of oscillation of the pendulum. The number of complete oscillations performed in one second is called its frequency.

92. Motion of a Simple Pendulum.—When the pendulum is at rest, the weight of the bob acts vertically downwards and passes through O, the point of suspension, so that the string is vertical. Suppose that the bob is drawn aside to the position B, so that OB makes an angle θ with OA, the vertical direction. At B the weight mg of the mass m can be resolved into two components one $mg \cos \theta$ acting along the string and in the direction BD, and the other $mg \sin \theta$ acting in the direction BE. The first of these components keeps the string taut and is balanced by the tension in the string. Hence the bob during its motion describes an arc of a circle of radius l, where l is the length of the pendulum. The second component is effective in producing in the mass m of the bob an acceleration $g \sin \theta$ in the direction BE which is evidently the tangent to the arc BA at B. On account of this acceleration the bob will come down along the arc BA whose centre is point of suspension, when it arrives at the lowest position A, it has acquired a velocity so that by virtue of its inertia it does not stop at A but continues to move along the arc AC on the side opposite to B. As the bob rises, however, the force due to gravity acting downwards now opposes the motion which accordingly becomes slower. Had there been no resistance of the air and friction at the point of support, the bob would stop on rising to a point C at the same height as B. It then descends again, passes through its mean position A and returns to the point B. It will thus continue to oscillate between the two points B and C for an indefinite number of times, all the vibrations being of equal extents and executed in equal intervals of time.

In practice, however, the energy of the pendulum slowly diminishes, a part of it being continually spent in overcoming the resistances to motion. The effect is that the amplitude of its oscillation gradually diminishes until the whole stock of its energy is exhausted, when it comes to stop at its initial position of rest at A.

If θ is small, the arc AB may be regarded as a straight line and

$$\text{Sin}\theta = \theta = \frac{\text{arc}}{\text{radius}} = \frac{AB}{l}.$$

The acceleration of the bob is given by

$$\text{Acceleration} = g \sin\theta = g\theta = g/l \times AB.$$

But AB is the displacement corresponding to θ. As θ decreases, the acceleration of the bob decreases and the displacement also decreases, the relation between the last two quantities being given by

$$\text{Acceleration} = g/l \times \text{displacement}$$

The bob of the pendulum therefore moves in a straight line, its acceleration being always directed towards the central position A and proportional to its displacement from A, for g/l is a constant. The motion is consequently a S. H. M. (see §35) and the period according to eqn. (26) is given by

$$\frac{\text{Acceleration}}{\text{displacement}} = \frac{g}{l} = \frac{4\pi^2}{T^2}$$

$$\text{or } T = 2\pi\sqrt{\frac{l}{g}} \quad \dots (67)$$

It is thus seen, that so long as the amplitude is small (θ not greater than 2° or 3°), the period of a pendulum is independent of it and also of the mass of the bob. This formula embodies all the laws of the simple pendulum and provides the most accurate means of determining g.

93. Laws of the Pendulum.—The oscillations of a pendulum are expressed by the following laws :—

LAW I.—LAW OF ISOCHRONISM—*The oscillations of a pendulum are isochronous i.e., effected in equal times, provided the amplitudes are small.* This law is perfectly true when the angle of displacement is not more than 2° or 3°. The uniform rate of motion of a clock depends on this property of a pendulum.

For, if l and g are constant, eqn. (67) indicates that T will also be constant

The law was first discovered by GALILEO. This was his first discovery made before he was twenty years of age, and while he was still a student of medicine at the University of Pisa. He happened one day to observe in the cathedral at Pisa the swinging of a bronze lamp hanging from the lofty roof and was struck by the fact that the period of oscillations of the lamp remained constant although the amplitudes were getting smaller and smaller. To make quite sure of this, he put his fingers on his own pulse and comparing its throbs with each swing of the lump, found that there was always the same number of beats to every swing. Following up this simple observation he discovered that a weight at the end of a cord will always take the same time to swing backwards and forwards, so long as the cord is of the same length and the arc through which the weight moves is small. This was the *beginning* of the pendulum, though at first it was only used by physicians to count the rate of a patient's pulse-beats. The merit of having first made the application of a pendulum to clocks is generally attributed to Huyghens.

Galileo discovered also the law of length which is given below.

Law II.—Law of Length.—*The period of oscillation of a simple pendulum varies as the square root of the length, i.e.* $T \propto \sqrt{l}$. Thus if the length of a pendulum is increased 4, 9 or 16 times, the period will be 2, 3 or 4 times respectively.

The length of a simple pendulum is, as already mentioned, the distance between O, the point of suspension, and the centre of the bob. In a physical pendulum this is given by the distance measured from

the point of suspension to the centre of gravity of the whole body undergoing oscillation. For example, when a pendulum is of the form represented in fig. 103, its length is the distance between O and G, where G is the centre of gravity of the rod, bob, screw etc., taken as a whole.

When a clock is going too slow or too fast, the length of its pendulum must be altered to regulate it. If the clock goes too fast, it means that its pendulum oscillates too quickly and the period must therefore be increased. This is secured by lowering the lens-shaped bob a little along the pendulum rod by means of the adjusting screw provided for the purpose. This procedure lowers the C. G. of the system thereby increasing the length of the pendulum.

Law III.—*The period of oscillation of a simple pendulum is independent of the mass and the material of the bob.* In other words, if the length of the pendulum remains the same, it does not matter whether the bob is heavy or light, is of wood or brass, or ivory or of any other material ; the time of swing will remain the same.

Law IV.—*The period of oscillation varies inversely as the square root of the acceleration due to the gravity at the place where it oscillates, i.e.,* $T \propto 1/\sqrt{g}$

It follows that if the value of g at a place is greater than that at another place, the period of oscillation of a pendulum will be smaller, *i.e.*, the vibrations will be quicker.

Thus a pendulum will oscillate quicker at the poles than at the equator.

Experimental Verification of the Above Laws :

Arrange an apparatus as in Fig. 104. Draw the bob aside taking care that the displacement from the vertical position is not too great. Then release the bob smoothly. It will begin to oscillate freely. Note by means of a stop-watch the total

time taken by the pendulum to perform some 20 complete oscillations. This time divided by 20 gives the period of oscillation of the pendulum. Repeat the experiment and measure the per od each time. It will be seen that the period remains the same in each case. This proves the truth of the first law.

Determine the periods of oscillation by varying the length of the supporting string. By means of a slide calipers measure the diameter of the spherical bob and hence find its radius. The length of the supporting string plus the radius of the bob gives the length l of the pendulum. Tabulate your observations so that one column contains the observed periods, while a second one contains the square roots of the corresponding lengths l. It will be found that $T/\sqrt{l}$ is constant. This verifies the second law.

Use bobs of different meterials in turn. Adjust the length of the string in each case so that the length of the pendulum *i.e.*, the distance between the point of suspension and the centre of the ball remains the same. Observe the period each time. The period will be found to be unaltered. This verifies the third law.

The verification of the fourth law cannot obviously be accomplished at the same place. The results of observations made by several observers at different latitudes show, however, that this law is also true.

94. Value of 'g' by the Pendulum.—The formula for the period of an oscillation of a simple pendulum provides an excellent and the best means of determining accurately the value of g, the acceleration due to gravity at a place. If we observe T and l, we have

$$T = 2\pi\sqrt{\frac{l}{g}}$$

whence $$g = \frac{4\pi^2 l}{T^2} \qquad \ldots \quad \ldots \quad \ldots \quad (68)$$

and from this we can calculate g.

The pendulum experiments have established that while g is a constant for all bodies at a given place on the earth's surface, it varies from place to place; in other words, the value of g varies with the latitude. It increases as one proceeds from the equator to

either of the poles. It is about 979 cm. per sec. per sec. at the equator and about 983 cm. per sec. per sec. at the poles. This diminution of g from the poles to the equator has already been considered (art 57).

The Value of 'g' below the Surface of the Earth.— Experiments with a pendulum show that the period of oscillation of a pendulum increases as we ascend higher above the sea-level, or descend into the earth as in a mine. It follows from eqn. (68) that the acceleration of gravity g diminishes with increase of height above and with increase of depth below the surface of the earth as in a mine. The first fact has been proved in art. 57. The diminution of g as we descend below the surface of the earth remains to be explained. It may be theoretically shown that a spherical shell exerts no gravitational force on any particle placed *inside* it. Consider a point P at a depth d below the surface of the earth as in fig. 106. Describe a sphere through P round O, the centre of the earth. Then the point P is inside the spherical shell (shown shaded in the figure) of thickness d whereas it is outside a sphere of radius $R-d$, where R is the radius of the earth. Since the shell exerts no force on a particle at P, the only gravitational pull that the particle experiences is that due to the sphere of radius $R-d$. The volume of this sphere is $\frac{4}{3}\pi\ (R-d)^3$ and the volume of the whole earth is $\frac{4}{3}\pi R^3$. Hence the mass of this sphere is $[\frac{4}{3}\pi(R-d)^3]\ /\frac{4}{3}\pi R^3$ *i. e.*, $(R-d)^3/R^3$, of the whole mass of the earth.

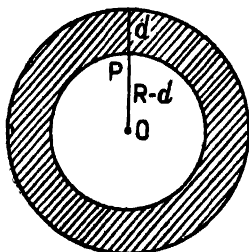


Fig. 106
Acceleration of gravity at a point inside the earth.

Hence the gravitational force attracting a particle of mass m towards the centre of the earth is given by

$$\text{Force} = G. \left[\frac{M(R-d)^3}{R^3}.\ m \right] \Big/ (R-d)^2,$$

where G is the gravitational constant and M, the mass of the earth

$$= G. \frac{Mm}{R^2}. \frac{R-d}{R}$$

Acceleration of gravity at P is given by

$$g' = G. \frac{M}{R^2}. \frac{R-d}{R}$$

$$= g. \frac{R-d}{R} \quad \ldots \quad \ldots (69)$$

where g is the acceleration of gravity on the surface of the earth.

This equation shows that the acceleration due to gravity diminishes as the depth d increases, *i.e.*, as we descend lower down into the earth.

95. The Seconds Pendulum.—A simple pendulum which makes half a complete oscillation in one second, is called a **seconds pendulum**. Hence putting T=2 secs. in eqn. (67) for the period of oscillation, we have

$$1 = \pi \sqrt{\frac{l}{g}}$$

$$l = \frac{g}{\pi^2} \quad \ldots \quad \ldots (70)$$

This gives the length of the seconds pendulum. Since g varies at different places on the earth's surface, the length of the seconds pendulum also is

not the same for all places. For an approximate value, putting $g = 32{\cdot}2$ ft. per sec. per sec. and $\pi^2 = 9{\cdot}87$, we have

$$l = 3{\cdot}26 \text{ ft.} = 39{\cdot}12 \text{ inches,}$$

and putting $g = 981$ cm. per sec. per sec. we get

$$l = 99{\cdot}39 \text{ cm.}$$

EXAMPLES :

1. A faulty seconds pendulum loses 9 seconds per day ; find the required alteration in its length, so that it may keep correct time. 1 day = 86400 seconds.

Since the pendulum loses 9 seconds per day, it beats $(86400-9)$ or 86391 times per day *i. e.*, in 86400 seconds : so that its time of half oscillation is 86400/86391 second (and not 1 second as it ought to be). Let l be its length

Then

$$\sqrt{\frac{l}{g}} = \frac{86400}{86391} \qquad \ldots \quad (1)$$

Let its length be changed from l to $l+x$ to make it keep correct time ; since, in that case, it becomes a true seconds pendulum, its time of half oscillation becomes 1 second. Hence

$$\sqrt{\frac{l+x}{g}} = 1 \qquad \ldots \qquad \ldots \quad (2)$$

From (1) and (2)

$$\pi^2\frac{l}{g} = \left(\frac{86400}{86391}\right)^2 \text{ and } \pi^2\frac{l+x}{g} = 1$$

Subtracting we have

$$\pi^2\frac{x}{g} = 1 - \left(\frac{86400}{86391}\right)^2$$

$$-\left\{1 + \frac{9}{86391}\right\}^2$$

$$\frac{18}{86391} \text{ approximately}$$

$$\therefore\ x = \frac{g}{\pi^2} \times \left(-\frac{18}{86391}\right) = -\frac{32 \times 7^2}{22^2} \times \frac{18}{86391}$$

$$= -0{\cdot}008 \text{ in.}$$

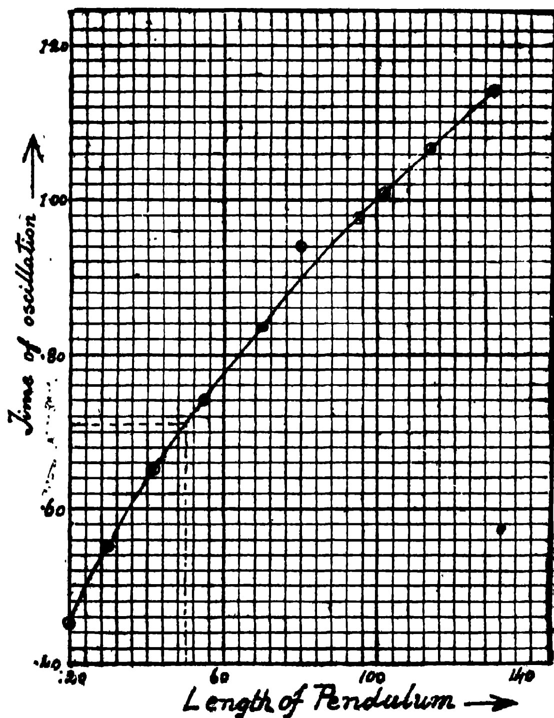


Fig. 107

Hence the length of the pendulum must be *shortened* by 0,008 in.

2 State the laws of the pendulum.

The following readings were obtained with a simp'e pendulum.—

Length	Time of oscillation	Length	Time of oscillation
20 cms.	·45 sec.	80 cms.	·94 sec.
30 ,,	·55 ,,	95 ,,	·98 ,,
42 ,,	·65 ,,	102 ,,	1·01 ,,
55 ,,	·74 ,,	115 ,,	1·07 ,,
70 ,,	·835 ,,	130 ,,	1·14 ,,

Represent by a graph the relation between length and time, and find from your graph the time of oscillation of a simple pendulum of length 50 cm. [*C. U.*—1915]

In the above graph

I small division along X- axis=4 cms.

I qmall division alongY- axis=·02 secs.

From the graph the time of oscillation of a simple pendulum of length cms =·71 secs.

Exercise IX

1. Two simple pendulums of lengths, 1 metre and 1·1 metre respectively start swinging together with the same amplitud : find the number of swings that will be executed by the longer pendulum before they are again swinging together (g=978). [C. U.—1909]

2. Describe in detail how you would test by means of pendulum experiments whether the acceleration due to gravity is the same for all substances [C. U.—1910]

3. State the laws of oscillation of a simple seconds pendulum. at a place where g is 981. 99·39 Cm

When a ball suspended by a string is made into a 'seconds pendu'um', does the actual length of its string equal the length of the equivalent simple pendulum ? If not, why ? [C. U.—1912]

4. State the laws of oscillation of a simple pendulum. Describe the effect of temperature on the period of oscillation of a compound pendulum. [C. U.—1918]

5. The laws of the simple pendu'um are summarized in the formula $t=2\pi\sqrt{l/g}$. Explain clearly the meaning of each symbol in the formula.

If the frequency of oscillation of a pendulum is 98 per minute at a place where $g=980$ cm. per sec., find the length of the pendulum. [C. U.—1916]

6. What is a Simple Pendulum? find the length of the Seconds Pendulum? at a place where $g=981$.

What is the exact meaning of the statement $g=981$?

Will a pendulum clock gain or lose when taken to the top of a mountain from the bottom? [C. U.—1917 ;' 19]

7. State the laws of a simple pendulum. How will you proceed to determine the 'g' of a place with pendulum? Give the practical necessary and state reasons.

What is the effect of the height above, or the depth below the surface of the earth, on the periodic time of a pendulum? Explain. [C. U.—1921]

8. State the laws of the pendulum. Will the period of vibration of a pendulum be affected, if it be taken to the top of hill? Give reasons for your answer. [C. U.—1924]

9. State the laws of a simple pendulum, and explain how they are verified. [C. U.=1928 ; 32]

10. A pendulum of length l loses 5 secs. a day. By how much must it be shortened to keep correct time? [C. U.—1932]

11. Explain why a given pendulum oscillates more quickly on the surface of the earth than in a deep mine.

CHAPTER XI

WORK, POWER AND ENERGY

96. Work.—Whenever a body on which a force acts, is displaced in the direction in which the force acts, **work** is said to be done *by* the force.

A horse drawing a cart along a rough level road does work ; an engine drawing a train does work. In each case, the body pulled on moves in the direction in which the force is exerted. Here work is done *by* a force.

But at the same time work is also done *against* some other force. Thus when a horse draws a cart, work is done against the frictional resistances of the ground ; in fact, had the ground been perfectly smooth, no work would have been done in drawing a body along its surface ; similarly, when a heavy mass is lifted up from the ground, work must be done against its weight, *i. e.*, the force of attraction due to gravity.

The work done *by* a force is measured by the product of the magnitude of the force and the displacement measured along the line of action of the force.

Let s be the displacement of the point of application of a force F in the direction of action of the force, then W, the work done *by* or *against* the force is given by

$$W = F.s \qquad \ldots \qquad (71)$$

When the displacement s takes place in a direction making an angle θ with the direction of the force, the component of the displacement in the direction

of the force is $s \cos\theta$. The work done by the force is given by

$$W = F.s\cos\theta \qquad \ldots \qquad (72)$$

The product $F.s \cos\theta$ may also be looked upon as the product of the displacement s and the component $F \cos\theta$ of the force in the direction of the displacement.

It should be noticed that no work is done by a force in a direction at right angles to that of the force, for a force has no component in a direction perpendicular to its own line of action. Also no work is done, when there is no displacement of the point at which the force acts. Thus when a man is unsuccessful in lifting a heavy weight, however hard he may try to do it, he does *no* work against the force of gravity. Moreover, in the expression for work, the time in which the displacement takes place, does not occur. Hence work done for a given displacement is independent of the time.

From a practical point of view, however, it is important to consider not only the amount of work done by a machine but also the time taken by it to do the work; in other words, the rate at which it is done. This rate is called the **power**.

Units of Work.—The unit of work is the work done by a unit force in displacing a body through unit distance in its own direction.

Just as there are four different units of force, so there must be four different units of work. These are :—

(i) the erg

(ii) the foot-poundal

(iii) the gram-centimetre and

(iv) the foot-pound.

(i) *The* **erg** *is the amount of work done by a force of one dyne in displacing a body through one centimetre in its own direction.*

(ii) *The* **foot-poundal** *is the amount of work done by a force of one poundal in displacing a body through one foot in its own direction.*

(iii) *The* **gram-centimetre** *is the amount of work done by a force of one gram-weight in displacing a body through one centimetre in its own direction.* It is therefore *equal to* the work done by gravity when a mass of one gramme falls freely through one centimetre.

(iv) *The* **foot-pound** *is the amount of work done by a force of one pound-weight in displacing a body through one foot in its own direction.* It is therefore *equal to* the work done by gravity when a mass of one pound falls freely through one foot.

The first two of these units are absolute units for they are constant under all circumstances. The last two are called gravitational units for they depend on the magnitude of the gravitational force exerted on unit masses. These are, therefore, not constant but varies from place to place. The first and third both belong to the C. G. S. system and the second and fourth belong to the F. P. S. system.

Since 1 gm.-wt. is equal to 981 dynes, and 1 lb-wt. is equal to 32·2 poundals,

1 gram-centimetre = 981 ergs,

1 foot-pound = 32·2 foot-poundals.

The erg is a very small quantity ; for example,

the work done in lifting a pound through 1 ft., *i. e.*, a foot-pound is given by

$$1 \text{ foot-pound} = 32{\cdot}2 \text{ foot-poundals}$$
$$= 32{\cdot}2 \times 30{\cdot}48 \times 13780 \text{ ergs}$$
since 1 ft. = 30·48 cm.
and 1 poundal = 13780 dynes
$$= 13{\cdot}6 \times 10^6 \text{ ergs.}$$

A larger unit, called a JOULE, is employed for industrial purposes and is equal to 10^7 ergs.

Hence

$$1 \text{ foot-pound} = 1{\cdot}36 \text{ joules}$$

97. Power.—*The rate at which an agent does work is called its* **power.**

The unit of power is the rate of doing unit work in unit time. The absolute unit of power in the C. G. S. system is the rate of doing one erg per second. In the F. P. S. system the absolute unit of power is the rate of doing one foot-poundal per second. These units are seldom used for practical purposes.

The practical units of power are (i) the horse-power (H. P.) and the watt. The **horse-power** was introduced by James Watt, the inventor of steam engines. Since his engines were frequently required to do the work hitherto performed by animals as in drawing a cart or in raising a weight, he thought it advisable to measure the power of his engines in terms of the rate at which a horse can do work. By an actual experiment he found that a horse can do 33,000 ft. lbs. of work in one minute. He, therefore, settled that

1 horse-power = 33,000 ft. lbs. per min.
= 550 ft. lbs. per sec.

The power of an average horse is less than this, about ⅔ of this rate, and the average man can work at only ⅒ of this rate nearly.

The **watt** is the rate of doing 1 joule, *i.e.*, 10^7 ergs

per second. A multiple of the watt called the kilowatt ($=10^3$ watts) is very frequently used for practical purposes. It is very nearly equal to $\frac{4}{3}$ H. P.

We have then

$$1 \text{ Watt} = 10^7 \text{ ergs per sec.} = 1 \text{ joule per sec.}$$

$$= \frac{1}{1{\cdot}36} \text{ ft. lb.}$$

$$= \frac{1}{1{\cdot}36 \times 550} \text{ H. P.}$$

$$= \frac{1}{746} \text{ H. P.}$$

or, 1 Horse power = 746 watts.

98. Energy.—We find from experience that in certain circumstances bodies are capable of doing work. The **energy** of a body is its capacity for doing work.

Kinetic Energy.—A body may possess energy by virtue of its being in *motion.* Thus every moving body is capable of working against resistances until it comes to rest. For example, the flying bullet, when it strikes a wooden target, penetrates a considerable distance into the wood working against the cohesive forces between the wood particles. A running stream is able to turn the wheel of a water-mill, the energy of which may be utilized in working grinding-machines, electrical generators (dynamos) and various other appliances. The energy of the wind in motion pressing against the sails of a boat may drive her in motion which would overcome the resistance of water offered to its passage.

This form of the mechanical energy is called the **kinetic energy.** Wherever we find matter in motion, be it solid, liquid or gaseous, it possesses kinetic energy. Further, the motion of a body may be a *translatory* motion (as in the case of a falling body, a

shot fired from a gun etc.), or a *rotatory motion* (as in the case of a spinning top) or a *vibratory motion* (as in the case of a vibrating pendulum, the vibrating particles of a sounding body etc.), or a motion of any other kind.

Potential Energy.—A body may also possess energy by virtue of its *position* relative to a body which attracts it. Thus when a weight has been raised up above the ground, it possesses energy due to its elevated position ; all the work spent on it in lifting it up is stored up in it , which is ready to be freed whenever the body is permitted to fall. For example, while falling, it can pull up a lighter body attached to it by means of a string passing over a pulley. In winding a clock, driven by a falling weight, work is spent on the weight to lift it up. As long as the weight remains at the elevated position, it possesses stored up energy which is expended during its fall in driving the clock and in overcoming the friction of the machinery. This form of mechanical energy is called **potential energy.**

Energy is measured in terms of work ; for when a body *does work against a force*, it loses energy, and if work *is done on it* by an external force, it gains energy. In either case, the loss or gain of energy is measured by the *work done by or against a force*. Hence the units employed in the measurement of energy are the same as those of work.

99. Kinetic Energy.—As has been stated in the last article *the kinetic energy of a body is the energy which a body possesses by virtue of its motion.* Instances of bodies possessing kinetic energy have already been given above.

Let us find an expression for the kinetic energy of a moving body :—

Suppose a body of mass m to be moving with a velocity u. We

have to find the kinetic energy possessed by it, i.e., the work it is capable of doing before it comes to rest.

The velocity of the body can only diminish if the motion is resisted by some external force and work is done by the body against it. Let P be this resisting force which we assume to be constant and let f be the negative acceleration produced by it. Then,

$$P = m\,f$$

Let us suppose that the body comes to rest after it has moved over a distance s. Hence from eqn. (18),

$$0 = u^2 - 2\,f\,s$$

$$\text{or } f = \frac{u^2}{2s}$$

Hence the kinetic energy of the body

= work done *by* the body against the force

= force × distance

$= P.s = m\,f\,s$

Or $\text{K. E.} = \frac{1}{2}mu^2$

Thus the kinetic energy of a body is given by half the product of its mass into the square of its velocity.

If the force P acting in the direction of motion increases the velocity of the mass m from u to v in passing over a distance s, we have,

$$v^2 = u^2 + 2fs$$

$$\text{Or } v^2 - u^2 = 2fs$$

The work done by the force

$= \text{P}.\ s.$

$$= mf\frac{v^2 - u^2}{2f}$$

$$= \tfrac{1}{2}\,m\,(v^2 - u^2)$$

$$= \tfrac{1}{2}\,mv^2 - \tfrac{1}{2}\,mu^2$$

= final K. E − initial K. E.

= increase of K. E. of the body.

In particular, if the body starts from rest, $u = 0$: the final gain of K. E. of the body is equal to the work done by the force, *i.e.*

$$\text{K. E.} = \tfrac{1}{2}\,mv^2 \qquad \ldots \quad (73)$$

Similarly, when the velocity of the mass decreases from u to v, the *loss* in K. E. is given by

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

It can be proved that the same expression for the kinetic energy holds good even when the resisting force is not constant.

If m is expressed in grams and v in centimetres per second, the K. E. is expressed in ergs. Similarly, when m is expressed in pounds and v in feet per sec., the K. E. is expressed in foot-poundals.

100. Potential Energy.—Some instances of bodies possessing this kind of energy have already been given. The **potential energy** may be defined to be *the energy possessed by a body (or a system of bodies) by virtue of its position or configuration.* It is measured by the work which a body is capable of doing in passing from its present position to some standard position ; or what is the same thing, by the amount of work done upon the body by a force in order to bring it from a standard position to its present position.

Thus when a spiral spring is compressed or pulled out, work has to be done against the elastic resistances offered by the spring to its disfigurement : this work is stored up as potential energy in the spring. If the spring be allowed to regain its previous form, it is able to do work against an opposing external force and so loses its energy. Thus the spring gains or loses energy by any change in its configuration.

In the same way, a bent elastic strip of metal, a coiled watch-spring, an extended India-rubber cord, an excited violin string, an extended bow string, a quantity of compressed air, all possess potential energy due to the *strain* or deformation in their materials, and all can do work when allowed to pass from their present configuration to the initial one. Strain energy is thus a form of potential energy.

The most important case of potential energy is, however, that in which a body possesses energy when it is lifted up from the surface of the earth. As a mass is raised upwards, work is done aginst its weight and the body gains in potential energy by virtue of its change of position relative to the earth. It is usually termed the *Gravitational Potential Energy* of the body. Water stored up in an elevated tank, a raised clock-weight, and an uplifted hammer, water-vapour floating in the air all possess gravitational potential energy.

It would have been proper and accurate, however, to regard the earth and the body on or near to its surface as forming *one system*, and that the system, made up of the earth and the body gains a quantity of energy equal to the work done in raising the body against the mutual attractions of the two, for the mass itself in the absence of the earth can have no potential energy.

It can be easily seen that if a body of mass m is raised through a *vertical* distance h at any place where g is the acceleration due to gravity, the work done against its weight is mgh. Therefore, the gravitational potential energy gained by the mass in this operation is mgh. Conversely, when a body of mass m falls through a vertical distance h at any place, the work done *by* its weight in falling is mgh which measures the *loss* of the potential energy of the body.

It should be noticed that h in the expression mgh is the *vertical displacement* of a body. If the displacement takes place in a direction inclined to the vertical, h should be replaced by the vertical component of the actual displacement.

The potential energy may also be stored by doing work against magnetic and electric forces. A

magnet attracts a piece of iron which tends to stick to the former. Hence, when they are separated, work is done and the system, consisting of the magnet and the iron, acquires a potential energy.

When a body is in stable equilibrium, its centre of gravity occupies the lowest possible position with respect to the earth (art 70). Any displacement of the body tends to raise its centre of gravity. Hence every such displacement tends to increase the potential energy of the body. Therefore, every state of stable equilibrium of a system (free from frictional resistances) corresponds to a *minimum of potential energy*.

101. Forms of Energy.—The energy which a body or a system of bodies possesses may exist in many forms but can always be put into either of the two classes, kinetic or potential or a combination of these two. The different forms in which energy may appear are :—

1. **Mechanical Energy** : the energy of a body falling from a height, of a body in vibratory motion etc.
2. **Heat** : the molecular kinetic energy in a body.
3. **Sound** : the energy of longitudinal wave-motion in a material medium.
4. **Light** : the energy of transverse wave-motion in the ether.
5. **Electrical Energy** and **Magnetic Energy.**
6. **Chemical Energy.**

Frequently, however, one form of energy is changing into another form. As a matter of fact all the phenomena in the universe are but cases of transformation of energy, which are almost endless in their variety.

102. Transformation of Energy.—In this article we shall consider some of the cases of transformation of energy.

Falling body.—When a body is at a height above the ground, it possesses gravitational potential energy. When it is allowed to fall, it gradually loses potential energy. The energy thus lost to the body is partly expended in overcoming the frictional forces of the air and partly in imparting kinetic energy to the body which is gradually increased. Just before striking the ground all the energy that the body possesses is kinetic. As the ball strikes the ground, its energy is transformed into heat (within the ball and the ground), sound and the mechanical energy of rebound, the latter two forms finally dissipating into heat in the surrounding space.

Vibratory motion.—A Pendulum.—The energy of a body in *vibratory motion*, *e.g.*, an oscillating pendulum bob, a vibrating violin string etc., is a combination of kinetic energy and potential energy.

The energy of vibration of a pendulum bob at any instant is what the bob possesses at that instant during its vibration, *in addition to* the energy it possesses when at rest. When the bob is at either of the extreme points, B or C in its path of vibration (Fig. 105), the energy of vibration is wholly gravitational potential energy for it is then momentarily at rest : and when the bob is at its initial position A, the lowest point in its path, the energy is wholly kinetic * ; at any other point in its path, the energy is partly kinetic and partly potential. As the bob proceeds towards the lowest point, it falls in height and therefore loses potential energy but gains kinetic energy ; conversely, as it passes from the lowest point towards either of the highest

* Assuming its position of rest at the lowest point A corresponds to zero energy level.

points, it loses kinetic energy and gains in potential energy. The total energy of the bob may be proved to remain constant excepting what is expended in overcoming the air resistance and also to a certain extent the friction in the suspension thread at the point of suspension. Due to this reason, the amplitude of vibration decreases gradually until the pendulum comes to rest. In order that the pendulum of a clock may continue to be in motion, energy is to be regularly supplied by the strain-energy of a clock-spring or by the potential energy of a raised clock-weight.

Molecular Energy.—In the *Molecular theory* of matter a quantity of matter may be considered as an aggregate of molecules held together by the action of the intermolecular forces (art. 106). A body may, therefore, possess **molecular potential energy** by virtue of its molecular configuration, *i. e.*, the relative positions of the molecules with respect to one another.

The *strain* energy or the energy due to the deformation in a body is thus the gain in its molecular potential energy. It is also known that the molecules in a body are not at rest but in an extremely minute and rapid vibratory motion, so that the body possesses **molecular kinetic energy** also.

Heat of a body.—The molecular kinetic energy of a body is associated with the *heat* of the body, which raises its temperature. When a button is rubbed on a table-cloth and then applied to the skin, it is found to be appreciably hot. The kinetic energy imparted to the molecules of the button in the process of rubbing appears as heat, conversely the extraction of kinetic energy from the molcules of a substance is accompanied by a loss of heat. Thus when the steam in an engine drives a piston, a part of the kinetic energy of the molecules of steam is used up in driving the piston and the steam is thereby cooled.

Change of state.—Again, when a body changes its state, *i.e.*, passes from the solid to the liquid form or from the liquid to the vapour form, it always absorbs a quantity of heat-energy which is called the **latent heat**, as it is not indicated by a rise of temperature of the body. This heat-energy is all expended in increasing the molecular potential energy that is necessary to bring about a change of state.

Wave-motion.—Light.—*Light* is beleived to be the transverse wave-motion in the medium ether through which it passes with a definite velocity and on reaching our eyes affects our sense of sight. These waves are electromagnetic in nature and hence the energy of light is composed of electric and magnetic energies. It has been calculated that to affect us as light the waves must be of lengths, within certain narrow limits, somewhere between 300 and 800 millionth of a millimetre.

Radiation.—But as we are familiar with the heating effect in the light from a glowing fire or from the sun, we are to infer that incandescent bodies send out other waves also, similar in kind to the light waves, but being of lengths outside the above limits they are incapable of exciting our sense of sight. The whole series of waves coming out is given the general name of **radiations** : the corresponding energy that is conveyed is called the **radiant energy.** The Radiant Energy is thus the form into which the heat energy in the radiating body changes when it is transferred into the surrounding medium as wave-motion.

Sound.—The energy of a tuning-fork or of a stretched wire set into vibration is similar to that of a pendulum bob. As the body vibrates, there is a periodic transformation of the *strain* energy and *kinetic* energy. The energy of the vibrating body is exhausted in giving rise to the sound-energy in the

surrounding medium and the heat-energy in the body in overcoming the internal friction.

Electric and Magnetic Energies.—If a rod of ebonite is rubbed with flannel, it acquires a new property of attracting bodies such as bits of paper towards it. It seems that the kinetic energy spent on the rod reappears in a new form. This new form of energy is called **electrical energy**. Similarly, when the blade of a steel-knife is rubbed with one pole of a magnet, it is found to acquire the new property of attracting small pieces of iron ; the form in which the energy reappears is called the **magnetic energy**.

These two forms of energy are closely associated with each other and are believed to be the manifestations of the different forms of energy of *strain and motion in the ether.*

Chemical Energy.—A molecule of a body may be supposed to be an aggregate of atoms held together by the *chemical affinity*. When water is decomposed by an electric current into its constituent elements,—hydrogen and oxygen,—the latter possess energy of chemical separation against atomic attraction, *i.e.*, **potential chemical energy,** the case being analogous to the energy of separation of a mass from the earth against the attraction of gravity. A small flame or an electric spark will suffice to rapidly convert the chemical energy of a mixture of hydrogen and oxygen into the kinetic energy of explosion. A loaded cartridge has a store of potential chemical energy which can, at any moment, be converted into the kinetic energy of the bullet.

Muscular Energy.—When a person lifts up a heavy weight by muscular exertion, the muscular tissue concerned in the action undergoes partial de-[illegible]tion. The chemical energy thus lost by the

tissue is transformed into other forms of energy. Thus a part appears as the heat-energy in the muscular portion and a part is expended in doing work on the body, which is stored up as the gravitational potential energy in the body.

Energy of Coal.—The solar energy i.e., the energy radiated by the sun, enables the plants and trees to decompose carbon dioxide present in the air into carbon and oxygen. Carbon is absorbed by them to maintain their growth while oxygen is let free into the atmosphere. Coal which is our chief fuel consists of carbon of wood which grew thousands of years ago and was then subjected to great pressures under the earth. Its combustion means the re-union of the carbon and oxygen atoms long separated. Thus coal and wood are stores of the potential chemical energy produced by the sun's light and heat. When they burn, their chemical energy is reconverted into heat and light.

The heat-energy obtained by the combustion of coal may again be changed by a locomotive steam-engine into the kinetic energy of the piston, of the driving wheel and then of the moving train. As mentioned before, a part of the mechanical energy reappears as heat in the various parts wherever friction is overcome. In the case of a stationary engine used in driving a dynamo, the heat expended by the engine may be transformed into the electrical energy which again may be utilized to do mechanical work *e. g.*, driving a tram-car or a fan, or to produce light in electric lamps or to produce heat in electric stoves and radiators.

Energy of Running Stream—To take another example of transformation of energy, vast quntities of sea-water are being daily evaporated under the action of the sun's rays, the vapour so formed, condensing into clouds in the higher regions of the atmos-

phere. This water comes down to earth in the form of rain which feeds the running streams and flows down to the sea, its place of origin. The energy of the running stream is thus derived from the potential energy of the clouds, of which again the source is the solar energy.

103. Conservation of Energy.—The examples considered in the last article point to the fact that the disappearance of any form of energy is always accompanied by the appearance of one or more other forms. Indeed, the results of many experiments made with the greatest care have led to the conclusion that there can be no case in which one kind of energy is absolutely annihilated without the appearance of another, nor any in which a form of energy appears *de novo* without the loss of another.

Thus a body or a system of bodies, may lose energy in one form, and gain an equal amount of energy in some other form or forms. Thus a body or a system of bodies A may do work on some other body or system of bodies B and thereby lose energy in some form, while the body or the system of bodies B gains energy either in the same form or in some other form or forms ; and the energy lost by A is equal to that gained by B. If A and B are regarded together, it is evident that the total amount of energy in the complete system must remain constant and connot be increased or diminished in any way.

This is known as the **principle of conservation of energy** and is a fundamental law in Physics. The principle is stated thus :

The total energy of any material system can neither be increased nor diminished by any action between the parts of the system, though it may be transformed into any of the forms of which energy is susceptible.

It should be noticed that the statement "by any

action between the parts of the system" implies that the system is to be regarded as isolated from the action of all bodies external to itself. It follows that the total quantity of energy present in the universe always remains the same.

It is also to be remembered that in any case where energy is transformed from one form to another, it does so according to a definite rate of exchange. Thus it has been established that the quantity of heat necessary to raise the temperature of 1 lb. of water through 1° F. possesses the same amount of energy as is required to lift 778 lbs. against gravity through one foot, *i.e.*, the mechanical equivalent of the unit quantity of heat is 778 foot-pounds.

104. Dissipation of Energy.—Although the total quantity of energy in the universe remains constant, so that the disappearance of energy in one form is always accompanied by the appearance of energy in some other form or forms, the energy available to man for the purpose of doing work is continually diminishing. This is known as **dissipation of energy.** To take an example, if a stone dropped from a great height, strikes the ground its kinetic energy disappears ; and the stone and the earth become warmer as the result of the impact. In a very short time the heat of the stone and the earth will be diffused among the surrounding objects. So far as we are concerned, the energy of the stone has been wasted.

In a heat-engine a large proportion of heat produced by the combustion of the fuel is lost in doing work against the friction within the machinery and is thereby converted into heat which warms up these parts.

In fact, in every transformaion of energy, a part of the energy becomes converted into uniformly diffused heat. Energy in this form is of no more use to

us for doing work. Heat can be expended in doing work only when it passes from a hotter body to a colder one; for example, from the boiler to the condenser in a steam-engine. If the temperature of a system is uniform throughout, no portion of the energy of the system is available for use.

As the total quantity of energy in the universe is a constant quantity, it follows from the above that the stock of energy available for work is steadily decreasing and that there must arrive a time when all the energy will be unavailable, the whole universe having become a uniformly hot and inert mass.

EXAMPLES :—

1. Prove that when a particle of mass m falls from rest at a height h above the ground, the sum of its potential and kinetic energies is constant throughout the motion,—if the frictional resistance due to air be neglected.

[*C.U—1932*]

Let v be the velocity of the particle when it has fallen through any distance x.

We have $v^2 = 2\,gx$.

Its kinetic energy at that instant is given by

$$\text{K. E.} = \tfrac{1}{2}\,mv^2 = \tfrac{1}{2}m \times 2\,gx = mgx$$

Also its potential energy at this height

$$= mg\,(h-x)$$

Hence the sum of its kinetic and potential energies

$$= mgx \times mg\,(h-x) = mgh$$

which is the potential energy of the particle at the height h forming its total energy.

2. If the clouds were three-fourths of a mile above the earth, and enough rain fell to cover half a square mile of the surface half an inch deep, how much work was done in raising the water to the clouds? From what did the energy come?

[*C. U—1922*]

Vol. of water of rain fall $= \frac{1}{2}$ sq. mile $\times \frac{1}{2}$ in.

$$= \tfrac{1}{2} \times (1760)^2 \times 3^2 \times \tfrac{1}{2} \times \tfrac{1}{12} \text{ cu. ft.}$$

But 1 cu. ft. of water weighs 62·5 lbs

$\therefore$ Mass of water of rain fallen $= \frac{1}{2} \times (1760)^2 \times 9 \times \frac{1}{2} \times \frac{1}{12} \times 62·5$ lbs.

$= 330 \times 1760 \times 62·5$ lbs.

This has been raised through $\frac{3}{4}$ of a mile above the ground.

$\therefore$ Work done $= 330 \times 1760 \times 62·5 \times \frac{3}{4} \times 1760 \times 3$ ft. lbs.

The source of energy is, of course, the heat of the solar radiation.

Exercise X

1. Define K. E. and P. E. Explain the terms "conservation and transformation of energy". Give examples.

2. What is meant by (i) an Erg (ii) a Watt and (iii) a Horse-power ? What is the relation between them ?

3. State the principle of conservation of energy. Illustrate the principle by taking some simple examples. [C. U.—1913]

4. A railway train is moving with uniform speed (*a*) on a level country (*b*) up-hill Explain how the energy supplied by burning coal in the engine is being expended in the two cases. [C. U. 1911]

5. It is said that most forms of the terrestrial energy, are derived ultimately from the sun. Explain the meaning of the statement, and discuss its truth with special reference to the energy of combustion of charcoal and of coal-gas, and the kinetic energy of a running stream. [C. U.—1912]

6. Define 'work' and energy.' Give simple examples of transformation of energy.

State also the principle of the conservation of energy. [C. U.—1916]

7. Distinguish between *work* and *energy*. A body falls under gravity and strikes the ground. Explain how the phenomenon supplies an illustration of the transformation of energy.

Does it also illustrate the principle of the conservation of energy ? How ? [C. U.—1917]

8. Distinguish between potential and kinetic energy with illustrations.

A railway train is going uphill with a constant velocity. What is the source from which the energy of the train is supplied ? Describe the various transformation of energy that go in this case. [C.U.—1918]

9. Explain clearly the meaning of the terms 'work' and 'energy'. Illustrate your answer by examples.

A body is projected upwards with a velocity of 64 ft. per second. Represent graphically its kinetic energy at any height during the upward journey [$g=32$] [C. U.—1919]

10. Explain clearly what is meant by energy of a body. State the *principle of conservation* of energy.

If clouds were one mile above the earth and rain fell, sufficient to cover one square mile at sea-level, ½ inch deep how much work was done in raising the water to the clouds ? [C. U – 1920]

11. Explain where the energy goes when you expend it in (*a*) winding up a watch, (*b*) lifting a stone from the floor and placing it on a shelf, (*c*) riding a bicycle uphill, (*d*) rowing a boat on a still pond, (*e*) rowing a boat up-stream. [*Pat.* U.—1921]

12. Define Work, Energy, and Horse-power, and distinguish between Kinetic and Potential energies.

Write a short note on the principle of the Conservation of Energy. [C. U.—1923]

13. What are kinetic energy, potential energy, and work ? Find the energy stored in a train weighing 250 tons and travelling at 60 miles per hour. How much energy must be added to the train to increase its speed to 65 miles per hour ? [C. U—1925]

14. What is meant by 'energy' ?

Describe suitable experiments to illustrate the following, and point out what they ultimately demonstrate :

(*a*) Conversion of mechanical energy.
(*b*) Conversion of electrical energy.
(*c*) Conversion of heat energy into luminous energy. [C. U.—1929]

BOOK II

PART VIII

PROPERTIES OF MATTER.

CHAPTER—XII

CONSTITUTION OF MATTER.

105. Different States of Matter.—Material substances are found to exist in three distinct states—*viz.*, the solid, liquid and gaseous.

A **solid** is a form of matter which has a *definite shape, volume and mass.* It does not require any lateral support to maintain its shape and resists any change of shape. Wood, stones, ice etc., are solid. The solids are more or less hard.

A **liquid** has a *definite volume and mass*, but *no shape* of its own. When at rest, it takes the shape of the vessel in which it is contained and maintains a free horizontal surface. Water, oils, alcohol, mercury etc., are liquids.

A **gas** has a *definite mass* but *no definite shape or volume*. It completely fills the vessel in which it is enclosed, whatever may be the volume of the vessel. Gases are continually tending to expand, *i. e.*, to occupy a large space. Hydrogen, oxygen, air, carbon dioxide etc., are gases.

Liquids and gases are comprised under the general name of **fluids**. A Fluid is a substance which flows when unsupported, for its particles are more or less mobile.

It may be found, however, that there are substances which are in a state intermediate between the solid and the liquid states. For example, sealing-wax and pitch are semi-solids which do not retain their forms when left to themselves. Again, liquids such as treacle, honey, syrup, tar etc., do not rapidly

assume a free horizontal surface. They are said to be *viscous*.

By the application of heat solids may be changed to liquids and liquids to gases. Conversely, the process may be reversed by lowering the temperature of bodies and increasing the pressure, if necessary. Thus ice when warmed becomes water which may be boiled to generate steam; steam on cooling changes into water which again when exposed to great cold, becomes solid in the form of ice.

106 Constitution of Matter.—Suppose that a quantity of matter is continually divided and subdivided into smaller and smaller parts. It will be found that up to a time the small particles of matter so obtained all retain the properties of the original matter of which they form part. But ultimately a certain stage will be reached when, on further subdivision, each of these particles will break up into still smaller ones which possess properties quite different from those of the original particle. This ultimate particle which is the smallest subdivision of a piece of matter retaining the same identical properties of the original matter is called a **molecule** of its own. The smallest subdivisions of a molecule are called **atoms**. Thus in the case of a piece of chalk the molecules are the smallest particles of chalk possessing all its properties unchanged. Each molecule of chalk can, however, be further subdivided into five atoms, one of calcium, one of carbon and three of oxygen. These latter possess properties quite different from the original chalk.

Thus a body is an aggregate of molecules, each of which is again supposed to be made up of a group of atoms held together by the inter-atomic forces called the *chemical affinity*. In physics, the molecule is regarded as the unit in the constitution of matter of a particular kind.

If the atoms constituting a molecule of a particular substance are all of the same kind then that substance is called an **element** and when the constituent atoms are different from one another, the substance is called a **compound**.

Size of the Molecules.—We have no definite idea as regards the actual '*form*' of a molecule but it is assumed that they are spherical. As to its *size*, it is clear that it is inconceivably small. Approximate calculations have been made by scientists based on different experiments. Thus it has been calculated that a cubic millimetre of water, which is about the size of a pin's head, would contain a number of molecules equal to 3×10^{19} approximately. Some idea as to the probable size of a molecule, considered as a small spherical particle, may be obtained from an illustration by LORD KELVIN, in which he states that if a drop of water were magnified to the size of the earth, the molecules in it would be about the size of the cricket balls !

Intermolecular spaces.—When a pressure is applied to a body or its temperature is lowered, the volume of the body is diminished ; conversely, the length of a rod is increased, when its temperature is raised. Again, when sugar is dissolved in water, there is no corresponding increase in the volume of the solution. Such facts are best explained by supposing that the molecules which build up a body are not in actual contact. There are **intermolecular spaces** *i.e.*, the spaces between the molecules, the extent of which may be altered by the application of external physical forces.

Intermolecular forces.—The molecules of a body are held together by means of forces, called **intermolecular forces.** Such a force is very strong when the distance separating two molecules is very small compared to their dimensions, but it is very small and

almost vanishes when this distance exceeds a certain value. Thus it requires a large force to separate one part of a piece of lead from its adjacent parts ; but if they are separated by even a very small distance, as when the piece of lead is cut by a knife, the two parts will not hold to each other even when they are pressed together by a very great force.

To counteract the effect of these attractive forces which, had they existed alone, would have pulled the molecules into the closest contact possible, so that no pressure could have compressed them closer, it is assumed that the molecules of a body are not at rest, but are in a state of *rapid motion*, due to which there is a tendency of the body to expand.

Thus the molecules in a body possess *molecular potential energy* which is due to their configuration in the building up of the body and *molecular kinetic energy* by virtue of their motion (see art. 102). The physical state of a body at a given temperature is the result of the balance between these two opposing tendencies : one, a tendency to become as dense as possible due to the attractive action of the intermolecular forces, and the other,—a tendency to expand in virtue of the motion of the molecules.

Solid.—In the *solid state*, the intermolecular forces are very powerful, so that the molecules of a solid body cling together with a great force. This enables the solid to have a definite *shape and volume*. It is due to these forces that a solid offers a resistance to any change in its shape or volume. Thus it requires an enormous force, about 9 tons weight, to break a steel rod of 1 cm. in diameter into two pieces. Yet the molecules in a solid are supposed to vibrate rapidly about practically the same positions with reference to the others.

Liquid.—In the case of a liquid, the molecular attraction is rather feeble but is still perceptible. A

liquid has, therefore, no definite shape but it takes the shape of the vessel in which it is contained. It yields to a force, however small, which tends to change the shape of the mass, and flows out in a direction in which it is free to move. The properties of a liquid are more fully considered later on.

Gas.—The molecules in a gas are supposed to be so far apart from one another that the intermolecular forces are negligibly small or practically absent. The molecules tend to go away in all directions independently of each other. In accordance with the theory, known as the *Kinetic Theory of Gases* it is supposed that the molecules of a gaseous body are moving about with a great velocity, and that they are colliding with each other and bombarding the walls of the vessel in which they are enclosed. This latter causes the *pressure* which the gas exerts upon the surface enclosing it. So all the molecular energy in a gas is of the form of the kinetic energy.

Electrons and protons.—Recent experiments in electricity have confirmed that an atom is not the ultimate indivisible particle of matter. According to the *Electron Theory*, as has been formulated by Sir J. J. Thomson, and advanced by Sir Ernest Rutherford, Prof. Niels Bohr, Sir Oliver Lodge, Dr. Soddy and others, the atom is supposed to be made up of two kinds of electrically charged particles called the *Protons* and the *Electrons*.

The electrons are all alike and are charged with negative electricity. The amount of charge on an electron is constant and is the ultimate unit of electricity. It can not be further subdivided so that any quantity of charge can be expressed as an integral multiple of the electronic charge.

An atom of any element consists of a central positive charge called the *nucleus* surrounded by a number of electrons revolving round it. Atoms of different elements differ in the number of revolving electrons and in the composition of the nucleus. The simplest atom is that of hydrogen. It consists of a single electron revolving round the nucleus which is called a proton.

The nucleus of any other atom is the combination of a number of protons and electrons bound together. Now the mass of an electron is very small being about $\frac{1}{1850}$ of that of a proton. Hence the mass of each atom may be supposed to be concentrated in its nucleus.

CHAPTER XIII

GENERAL PROPERTIES OF MATTER

We have remarked in art 2 that matter is the stuff or the material of which bodies are composed, and which is perceptible by us through our senses. Matter is best defined, however, by the enumeration of its essential properties.

Properties that are found in common in all the states of matter, whether the solid, liquid or gaseous, may be called the **general properties of matter** *e.g.*, extension, impenetrability, inertia, gravitation, divisibility, porosity, elasticity and density. But those properties that are found in a particular state or states of bodies are called special properties. Thus there are special properties of solids, of liquids and of gases. These will be taken up later on.

107. Extension.—Every piece of matter must occupy some definite space. This property of matter is called the **extension.**

Extension regarded in one direction gives a *length*; in two directions, a *surface*; in three directions, a *volume.*

108. Impenetrability.—Two pieces of matter cannot occupy the same space at the same time. **Impenetrability** is the property in virtue of which a piece of matter occupies a space to the exclusion of all others. If a block of iron is immersed in water, the liquid has to move away to make room for the immersed body.

Expt. 18. Put a small bottle in water with the mouth downwards. It will not be filled with water till the mouth is so turned that the air in the bottle can escape.

The above simple experiment proves that air is impenetrable.

The term impenetrability is, however, not to be taken in the ordinary sense that one body cannot penetrate into another. A nail can be driven into a block of wood. Water poured upon a heap of sand disappears quickly. If 50 c.c. of alcohol is mixed with 50 c.c. of water, the volume of the mixture is less than 100 c.c. The true explanation, in all these cases is afforded by the existence of the intermolecular spaces within the body (see art. 105). Thus the molecules of wood are thrust aside to make a space large enough to admit the substance of the nail. In the case of water disappearing in sand, the water does not penetrate into the substance of the sand itself, but simply fills the spaces between the grains.

In fact, impenetrability and extension are not two different properties of matter independent of each other, but are merely two expressions for the one and the same thing.

109. Inertia.—Matter cannot, of itself, change its state of rest or motion. Inertia is a characteristic property of matter and is defined in Newton's first law of motion (see art. 45). It is the property in virtue of which a body *i.e.*, every form of matter continues in its state of rest or of uniform motion in a straight line, unless it is acted upon by an external impressed force to change that state.

Numerous illustrations of the inertia of bodies have already been given in art. 45

110. Gravitation.—Every piece of matter in the universe has the power to attract every other piece of matter; this is a fundamental property of matter. The force of attraction exerted mutually between any two pieces of matter is called **gravitation.**

The amount of this attraction depends on the quantity of matter in each and their distance from each other; it is greater when bodies have larger masses than when they have smaller ones, and also when they are nearer than when they are more remote.

The Law of Gravitation has already been discussed in art. 53.

111. Divisibility.—It is the property, in virtue of which a body can be divided into extremely small parts.

Bodies can be subdivided into smaller parts by mechanical methods, such as cutting, swing, filing, grinding etc. A piece of chalk is divided into a large number of detached particles when one writes on a black-board with it. Gold can be hammered into leaves so thin, that 300,000 of them come to the thickness of 1 inch. Glass, platinum and quartz have been drawn into fibres, so fine as to be quite invisible.

Very fine divisions are frequently effected by means of solution. A drop of carmine will colour a litre of water perceptibly red. It is calculated that a drop of blood which may be held on the point of a needle would contain about a million of blood-corpuscles floating in a colourless liquid, called the serum.

Still greater is the divisibility of odoriferous substances such as volatile essences, camphor, musk etc. The tenth part of a grain of musk is found to perfume a large room for years together and yet lose but little of its weight.

There is, however, a limit to the divisibility of matter. In the process of subdivision there must come a stage when a further subdivision will break up the ultimate particle, called the *molecule*, of the given kind of matter into its constituents, components, which are called atoms (see art. 106).

112. Porosity.—In a body the small particles of which it is composed are not in actual contact but they leave spaces or interstices between them. This is expressed by saying that bodies are porous.

Expt. 19. Take a lump of dry chalk and weigh it carefully. Next place the chalk in water for a few minutes. Air-bubbles will be seen to escape from the pores of the chalk, being displaced by the water particles coming in. Now weigh the wet chalk and notice that its weight has increased. Then increase in weight is due to the weight of the water that has filled the pores driving out air from within.

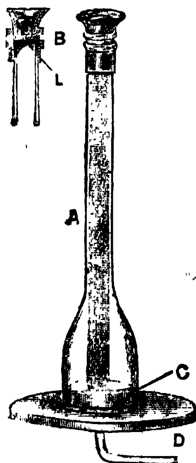


Fig. 108
Mercury rain.

Pores are to be distinguished from the intermolecular spaces. The latter are never directly perceptible and are so very small that the surrounding molecules remain within the spheres of each other's attraction. Contractions and expansions of bodies resulting from changes of temperature are explained by the existence of these intermolecular spaces. *Pores* are all actual cavities or holes which are sometimes distinctly visible under the microscope. They are too large for the intermolecular forces to act.

Fine pores exist in wood, cork, sponge, unglazed earthenware, paper, leather, India-rubber, pitch, cloth, skin etc. The existence of pores in leather is demonstrated in the following experiment, known as the *Mercury Rain.*

Expt. 20. A long glass tube A (Fig. 108) is provided with a brass cup B at the top. The bottom of the cup

consists of a thick piece of leather L. The lower part of A is wide enough to admit a cup C to be placed within it.

A circular disc D is provided at its centre with a tube connected to an air pump. Place A on the disc D and apply a little grease all round the base of A. Pour mercury to partially fill up the cavity B at the top. Work the pump to reduce the pressure of the air within the tube. A shower of mercury is produced within the tube, as mercury is forced through the pores in the leather by the greater pressure of the atmosphere outside. The vessel C is used so that mercury may not fall on the pump-disc.

In filtration, a liquid is freed from the particles suspended in it by allowing it to percolate through charcoal or carbon. Chemists use the filter-paper for this purpose in the laboratory. Blotting-paper is used for soaking ink : powders of chalk may also be used for this purpose. Deep-well water is generally clear as it is filtered on its way through thick strata of the earth. It is to be noted that a filter is of no use to keep back the *dissolved* impurities ; for example, salt-water when passed through a filter retains its saline taste.

The inter-penetration of two liquids, such as alcohol and water, is often chosen as an example of porosity in liquids.

113. Elasticity.—Elasticity is the property by virtue of which a body is able to resist a change of volume or of shape, and is able to resume its original volume or shape when the force that caused the change is removed, provided that the change has not exceeded a certain limit depending on the particular material of the body.

If a piece of India-rubber is squeezed and then let off, it returns to its original shape as soon as the force is withdrawn.

When a force or a system of forces acts on a body which is not free to rotate, or to move bodily, a displacement of the particles relative to one another

occurs thus causing a change of shape or of volume. Such a change in the shape or volume of the body is called a **strain**. Quantitatively strain is measured as the deformation per unit length or volume. It is, therefore, a pure ratio and as such has no dimensions.

In general, when a body is strained by the application of external forces, it resists the strain by setting up forces within the material, which not only oppose the change, but tend to restore the displaced particles to their original positions. The restoring force generated in the body due to the strain is called a **stress**. Stress is measured as the force per unit area of the section on which it acts so that the dimensions of stress are $[M]\,[L]\,[T]^{-2}/[L]^{-2}$, *i.e.*, $[M]\,[L]^{-1}[T]^{-2}$.

A body is able to offer this elastic resistance, when the strain is small and does not exceed a certain limit, called the **elastic limit** for the material of the body. Since within the elastic limit the stress generated within the body is equal and opposite to the external force producing the strain, the latter also is often called the stress. Hence we say that a stress applied to a body produces a strain.

ROBERT HOOKE established experimentally that if a body is strained *within its elastic limit*, the strain produced is directly proportional to the stress applied,

$$i.e.,\quad \frac{\text{Stress}}{\text{Strain}} = \text{a constant} \quad \ldots \quad (74)$$

which is called the **modulus of elasticity** of the material of the body for the particular kind of stress or strain under consideration. *The modulus is defined as the stress that is required to produce a unit strain.* The above relation is known as **Hooke's Law**.

Strains produced by the applied forces may be

divided into two classes, *according as they consist of a change in volume only or a change in shape only*. The former is called the **volume strain**; while the latter, the **shearing strain** or simply a **shear** (see art. 120).

Elasticity of volume is possessed by bodies in all the three states of matter. The modulus concerned can be expressed in symbols as follows :—

If a uniform pressure of p dynes per sq. cm., acting normally to the surface of a substance, be applied to reduce the volume V to $V-v$, then

the stress $=p$ dynes per unit area,

and the volᵗ strain = deformation produced per unit volume

$$= v/V$$

$$\therefore \text{ The modulus of volume elasticity } = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{p}{v/V} \quad \ldots \quad (75)$$

Illustrations of the volume-elasticity in solids are numerous. Thus corks pressed into the necks of bottles fit tightly. Elasticity of horse-hair, feather, wool, cocoanut fibres etc., is made use of in pillows, seats, cushions etc.

A liquid offers a very great resistance to a change in size, but almost no resistance to any change in shape; in other words, a liquid possesses a high modulus of the elasticity of volume, but is devoid of the elasticity of shape. This forms an important difference between a solid and a liquid.

A gas, like a liquid, has the elasticity of volume but no elasticity of shape.

Dimensions of a modulus of elasticity are the same as those of a stress for strain is of zero dimension (see eqn. 75). Hence the dimensions of a modulus of elasticity are $[M]\,[L]^{-1}\,[T]^{-2}$.

114. Density.—We say that gold is heavier than silver. Naturally, we mean that volume for volume, the mass of gold is greater than that of silver. The heaviness of a particular material, as we ordinarily mean by it, is therefore, dependant on the mass per unit volume of the material. This latter quantity is called **density**. The dimensions of density are $[M]/[L]^3 = [M]\,[L]^{-3}$.

Thus if a body of mass m occupies a volume v, its density P is given by

$$P = \frac{\text{mass}}{\text{volume}} = \frac{m}{v} \quad \ldots \quad \ldots \quad (76)$$

Different substances may contain very different quantities of matter in the same volume. Thus the mass of 1 c. c. of pure water at 4°C is 1 gramme, that of 1 c. c. of copper is about 8·9 gms., that of silver 10·5 gms., that of lead about 11·3 gms., that of gold about 19 gms. and so on.

Exercise XI

1. Enumerate the general properties of matter.
2. Find the volume of a block of lead weighing 100 gms.
3. A rectangular block of wood whose volume is 40 c.c. weighs 200 gms. Find the density of wood.

CHAPTER XIV

PROPERTIES OF SOLIDS

115. Hardness.—When one body can be made to scratch another and cannot be scratched by it, the former is said to be *harder* than the latter. Thus glass is harder than lead. Hardness is a relative property; a body which is hard with reference to one body may be soft with reference to others. Diamond is the hardest of all substances, for it scratches all of them, but is scratched by none.

The degree of hardness of a body is expressed by referring to a scale of hardness. The one generally employed is *Mohr's scale of hardness*, and is as follows, in which the substances are arranged in the order of increasing hardness :—

SCALE OF HARDNESS

1.	Talc	6.	Felspar
2.	Rock-salt	7.	Quartz
3.	Calc-spar	8.	Topaz
4.	Fluor-spar	9.	Sapphire
5.	Appatite	10.	Diamond

Thus the hardness of a body which would scratch felspar but be scratched by quartz lies between 6 and 7.

The pure metals are softer than their alloys. Hence for jewellery and coinage, gold and silver are alloyed with copper to increase their hardness.

116. Brittleness—This is the property due to which a body breaks down easily under the action of an external force, e. g., a blow from a hammer. Thus ice, glass are very brittle substances ; lead, copper are not.

By heating and cooling suddenly many substances, especially steel, become very hard though they become brittle at the same time. The process is called *tempering*. By the process of re-heating and slow cooling, which is called *annealing*, steel, glass etc., can be made less brittle.

117. Malleability.—It is the property possessed by some solids of being beaten into thin sheets. Pure gold is extremely malleable. Leaves of gold are obtained which are so thin that 3,00,000 of these, when superposed, become an inch thick only.

Malleability increases quickly with the rise of temperature, thus iron is easily forged when hot but not when cold.

118. Ductility.—It is the property of the solids of being drawn into fine wires.

Platinum is the most ductile of all metals. It can be drawn out into wires so thin that 140 of such wires placed side by side, have the thickness of a silk-fibre. Gold, silver, copper, iron, etc., can also be drawn into wires by means of a machine, called the *draw-plate*, in which the rod of a metal is pulled through a number of successive holes, each a little smaller than the last, bored in a plate of steel. Lead which is very malleable, is the least ductile.

Extremely fine wires of glass, quartz, etc. can be drawn when these are softened by the application of heat.

119. Elasticity.—A solid possesses both the elasticity of volume and the elasticity of shape.

Volume Elasticity or Bulk Elasticity.—When a solid is strained in such a way that it suffers a change in volume only without a change in shape *e. g.*, when a sphere is strained into another sphere, the elasticity concerned is known as **volume elasticity or bulk elasticity.** It has been seen in

art. 113, that the modulus of volume-elasticity which is generally denoted by k, is given by

$$k = \frac{p}{v/V} = \frac{pV}{v} \qquad \ldots \quad (77)$$

To strain a body in the above way is rather difficult. The applied stress must be a uniform pressure acting normally all over the surface of the body. Pressure may be applied in this way by immersing the body in a suitable liquid and then applying pressure to the liquid for the pressure on a liquid is transmitted equally in all directions and acts everywhere at right angles to the surface (See art 182).

Elasticity of Shape or Simple Rigidity—The *elasticity of shape* or as is more generally called, the **simple rigidity**, is the property in virtue of which a solid offers resistance when it is so strained as to suffer a change of shape or form without a change of volume. When a cylinder is subjected to a twist or torsion round its axis about one end, the other end being fixed, a shearing strain is produced.

Let ABCD (Fig. 109) represent the section of a rectangular block fixed at the lower surface BC to a horizontal bed. If a horizontal force P is applied tangentially along the upper surface AD, the block will be deformed so that its new shape will be A′BCD′, the surface AB being displaced laterally through a distance x. Every plane in the block is displaced relatively to the plane below it in the direction of the applied force but the height and consequently the volume remains unaltered. An idea of the nature of deformation may be obtained by piling a pack of cards in a

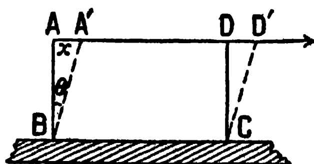


Fig. 109
Shearing strain

rectangular block as in fig.110 (a). and then displacing them as in Fig. 110(b).

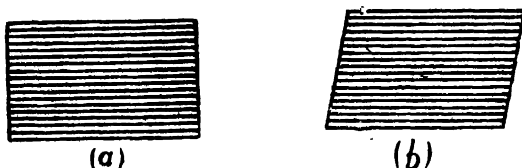


Fig. 110

Pack of cards in a rectangular block.

Pack of cards displaced with respect to one another.

The deformation produced in the above case is a pure change of shape. It is therefore called a shearing strain or simply a shear and is measured by the ratio $\frac{x}{l}$ where l is the length of the face AB. This ratio is the lateral displacement between two horizontal surfaces, unit distance apart. In all practical cases the displacement x is very small in comparison with l so that the ratio $\frac{x}{l}$ is equal to the angle θ, which is called the angle of shear. Hence,

$$\text{Modulus of simple rigidity} \quad \frac{\text{Stress}}{\text{Strain}} \quad \text{(Art. 114)}$$

$$= \frac{P/S}{x/l}.$$

where S is the area of the surface AD. This is also called *rigidity modulus* and is denoted by n. Therefore,

$$\frac{P/S}{\theta}.$$

The changes in the shape of a solid body may be brought about by

(i) tension, as in a rubber chest expander
(ii) flexure, as in bows or the strings of a harp
(iii) torsion or twist, as in a galvanometer suspension.

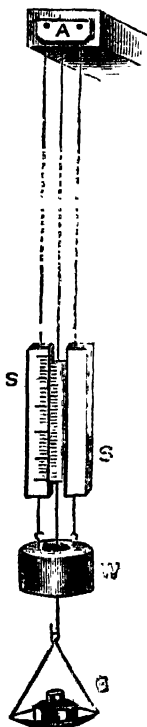


Fig. 111. Determination of Young's modulus.

(i) **Young's Modulus.**—If a rod or wire of uniform cross-section fixed at one end is stretched by applying a force at the other end in the direction of its length, the strain produced involves a change of both volume and shape. In this case the stress is measured by the force per unit area and the strain, by the change of length per unit length and the ratio of stress to strain is called **young's modulus.**

If P be the force which acting on a wire of length L and of cross section a stretches it by a short length l, then

the strain = elongation per unit length $= l/L$

and stress = force per unit area

$$= \frac{P}{a} = \frac{P}{\pi r^2}$$

if the cross-section is circular and of radius r, then the Young's Modulus is given by.

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{P}{\pi r^2} \cdot \frac{1}{l/L}$$

$$= \frac{PL}{\pi r^2 l} \quad \ldots (78)$$

The stress and strain involved in such cases of

longitudinal deformation are sometimes called **tensile stress** and **tensile** or **longitudinal strain**.

The elongation produced in a wire by a tensile stress is accompanied by a contraction in directions at right angles to the length of the wire. When the wire is of circular section, the ratio of the reduction in diameter to the original diameter is called **contractile strain** or **lateral strain**. So long as the longitudinal extension is small, the lateral contraction is directly proportional to the longitudinal extension. The ratio of the contractile or lateral strain to the tensile or longitudinal strain is called **Poisson's Ratio.**

Experimental Determination of Young's Modulus :—The value of the Young's Modulus for the material of a wire can be experimentally determined with the help of an arrangement shown in fig. 111. The wire is fixed at one end to a stout beam overhead and carries a vernier and a scale pan B attached to the other end. Two other wires of the same material are supported from the same beam on both sides of the first wire and carry two brass pieces S and S, one of which is graduated with a fine scale. These two side wires are kept stretched by a tubular weight W as in the figure. The vernier piece slides smoothly between S and S with its graduations facing against the scale engraved on one of them.

To begin with, the central wire is stretched by a suitable weight to free it from kinks. The length of the wire from the point of suspension to the point at which the vernier is attached is then measured. This gives L.

The diameter is then measured by means of a screw gauge and hence the radius r is known.

A known weight is then placed on the scale pan

and the elongation l produced by it is measured by means of the vernier. If m is the mass of the weight, the downward force is mg and the tensile stress is, therefore, equal to $\frac{mg}{\pi r}$ Thus the Young's Modulus is given by

$$Y = \frac{mg}{\pi r^2} \cdot l \qquad \ldots \quad (79)$$

The experiment is repeated by placing different

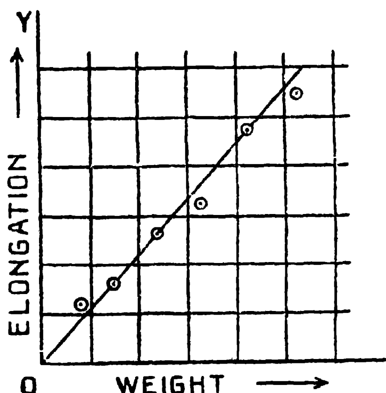


Fig. 112.

weights on the scale pan and noting the elongation in each case. A graph may then be drawn with the weight as abscissa and the elongation as ordinate. It will be seen that this graph is a straight line (Fig. 112) showing that Hooke's law is obeyed, *i. e.*, the stress is proportional to the strain. From the graph, the elongation corresponding to any suitable weight is read off and the Young's Modulus is then calculated from the above equation.

By subjecting wires of different materials to the

above treatment, their elastic constants can be compared. Thus it is found that for the same elongation if a steel wire requires 10 kilograms, an exactly similar wire of copper requires 6 kilos; that of brass 4·5 kilos and that of platinum 8 kilos. Steel is, therefore, about 1·7 time as elastic as copper and Platinum is nearly twice as elastic as brass.

In the above experiment it will be seen that unless the wire is stretched beyond the limit of elasticity for the material, it reverts to its original length. But even within the elastic limit, when the applied force producing a strain is allowed to act for a long time, the wire does not return to its exact original condition quickly, but takes time to do it; the wire is said to have acquired an **elastic fatigue**.

If the load on the wire be gradually increased, a stage is reached when the wire breaks down. The weight required for this, is called the **breaking weight** and is the measure of the **tenacity** or the tensile strength of the material of the wire. The tensile strength of the same material is directly proportional to the cross-section. The breaking stress for a cross-section of 1 sq. mm. is 34 kilos for brass, 30 kilos for copper, 60 kilos for iron and 30 kilos for steel.

Hooke's law is also found to hold in the case of elongation of the spiral wires. Hence these may be used for the purpose of measuring weights as in spring-balances, Jolly's spring, etc.

(ii) The **elasticity of bending** or **flexure** is seen in watch-springs, bows, carriage-springs, spring-balances etc. When an elastic flat piece, such as a steel plate, is bent, it is obvious that the fibres on the convex side are lengthened and put in tension, while those on the inner side are shortened and compressed. The laws according to which bending takes place in the elastic beams are of considerable interest

to the architects. A beam is much less easily bent when laid on its edge than when laid on its face. This is the reason why the beams of the roof of a house are placed on their edges.

Elasticity of steel is utilized in the carriage-springs, the buffer springs of railway carriages, the clock and watch springs, spring-balances, spring dumb-bells, dynamo-meters, etc.

(iii) The **elasticity of torsion** or **twist** is brought to action when a wire or a thread is twisted from its original configuration.

Expt. 21. Take a ball or a cylinder and support it from a long and thin wire, the upper end of which is fixed. Attach a strip of paper with a little gum at the bottom of the ball to act as a pointer. Rotate the ball and they leave it. The ball returns to its initial position after a few seconds.

This property of the torsional elasticity of a wire is utilized in the physical laboratory as a delicate means of measuring the magnetic and electric forces *e.g.*, in Torsion-balances, Vibration Magnatometers, etc.

Different solids present different degrees of elasticity. Glass, steel, ivory, marble are highly elastic bodies; while substances like lead, clay, fats, etc. possess scarcely any elasticity and are called *plastic* substances.

India-rubber is commonly taken as a highly elastic substance ; but the real fact is that it has a wide limit of elasticity. It may be stretched to twice or thrice its length, and still it regains the original length when the force is removed. Glass is much more elastic than India-rubber but its elastic limit is much narrower.

120. Cohesion.—From the various physical properties of solids, viz., hardness, elasticity, ductility, tenacity, etc. we are lead to suppose that the molecules of a solid cling to one another with forces

that are of great magnitude at inconceivably small distances. The attraction exerted by the molecules is called **cohesion** and the intermolecular forces are called the *cohesive forces.* The variation in the physical properties possessed by different solids depends upon the difference in the magnitudes of the cohesive forces existing between their molecules.

Cohesion is also present in liquids. It acts between the molecules of water or of any other liquid and causes them to join and form drops when the liquids are in small quntities. This is seen in raindrops, in dew-drops on the leaves of plants and in small globules of mercury (see also art 154). In large masses of liquids the force of gravity preponderates over those of cohesion ; hence these have no shape of their own, but take the shape of the vessel in which they are contained.

Adhesion.—Attraction is also exerted between the molecules of different kinds ; for example, between those of water and a finger or a glass rod dipped in it. To this has been given the name of **adhesion.** There is no real difference between cohesion and adhesion ; the former is often restricted to the attraction existing between the molecules of *like* kinds *i.e.*, to the molecular attraction inside a body, while the latter to the attraction between two *different* bodies with there surfaces in contact.

Glue sticking to wood, mortar to bricks, nickel plating to iron, copper to zinc in a solder, *etc.* are instances of adhesion and cohesion ; good adhesion is secured by applying the cement or solder in liquid condition, and cohesion is brought into play when the liquid is dried up. The particles of a crayon are held together by cohesion, while they stick to a blackboard due to adhesion.

It is due to adhesion again that two liquids mix with each other *e.g.*, alcohol and water, or that a

salt or sugar dissolves in water. Oil and water have no adhesion for each other and hence they do not mix. Resin has no adhesion for water and therefore does not dissolve in it. On the other hand, heat helps solution as a rule, because it lessens the cohesion within the soluble substances ; for the same reason, powdering a solid makes it dissolve more readily.

Exercise XII

1. Explain and give illustrations of the different kinds of elasticity possessed by a solid.

2. If a kilogram stretches a wire of 1 *mm.* in diameter by 0·5 *mm.*, how much will a wire of the same length but of twice the diameter be stretched by 8 kilograms ?

3. State Hooke's Law and define 'Young's modulus of elasticity.'

How would you determine Young's modulus for a steel wire ? (C. U.—1932)

4. State how you may, by use of Hooke's Law and a 20 lb weight, make the Scale for a 82 lb spring balance. (C. U.—1936)

5. Define coefficient of elasticity.

A mass of 20 kgs. is suspended from a vertical wire 600·5cms. long and 1 sq. mm in cross-section. When the load is removed, the wire is found to be shortened by 0·5 cms. Find Young's Modulus for the material of the wire.

(C. U.—1938)

CHAPTER XV

LIQUIDS

We shall now enter into the study of a particular branch of Physics called Hydrostatics. It deals with the equilibrium of liquids and gases and with the pressure they exert, whether within their own masses or on the sides of the vessels in which they are contained.

121. Fluids.—A **fluid** *is a substance which yields to any continued shearing stress, however small.* In other words, a fluid does not possess elasticity of shape while a solid does. But a fluid, like a solid, will resist a force tending to reduce its volume and will recover that volume when the force is withdrawn: thus it is said to have volume-elasticity. Hence the essential difference between a solid and a fluid is that while a solid possesses both elasticity of volume and shape, a fluid possesses only volume-elasticity.

It has been seen that the term *fluid* includes both the liquids and gases. A gas, however, differs essentially from a liquid in its *indefinite compressibility* and its power of *indefinite expansion.*

The **compressibility** in liquids is so small that for a long time liquids were regarded as incompressible substances. The first person to prove with an exact measurement that liquids are compressible was OERSTED, a Swedish physicist (1822). The instrument that he used is called the *Piezometer*. By means of this apparatus, he proved that all liquids are really compressible under *very great* pressures but only to a very slight degree; thus a pressure, equal to about 200 times that of the atmosphere, will reduce the

volume of a quantity of water by $\frac{1}{100}$th. part only. The same pressure will diminish the volume of a mass of mercury by $\frac{1}{1200}$th. part; of ether by $\frac{1}{10}$th. part. As soon as the pressure is removed, the liquid regains its original volume. But in the case of a gas the slightest increase in pressure will at once cause a corresponding change in the volume (Chap. XXI).

Again, if a quantity of a gas be enclosed in a vessel and the vessel be enlarged to any extent, it is found that the gas will still occupy the whole vessel: this is certainly not true of a liquid.

A **liquid** is *a fluid, the volume of a given portion of which never exceeds a definite amount, no matter to how large a space it has access, or how small the pressure to which it is subjected.*

A **gas** is *a fluid, a given portion of which can be made to expand indefinitely by increasing sufficiently the space to which it has access.*

122. Fluid Pressure.—The fundamental property of a fluid is that when *in equilibrium*, the force which it exerts on any surface, with which it is in contact, is *at right angles* to that surface. This can be easily seen by experiments. If a vessel is filled with any fluid, say water, and if a hole is made on the surface of the vessel at any point it will be seen that the water rushes out in a direction at right angles to the surface at that point. This proves that the fluid exerts a force on the surface of the containing vessel which is everywhere normal to the surface.

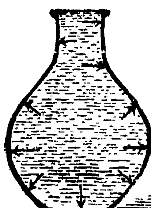


Fig- 113
Thrust on the sides of a flask

Thus in a flask of the shape shown in fig. 113

which is filled with a fluid (water, air, etc.), the force will be directed as shown by the arrow-heads in the figure.

123. Pressure at a Point.—The pressure at a point due to a fluid is the force exerted by the fluid on *unit* area surrounding that point.

If the pressure P exerted by a fluid is uniform over a surface of area α, the pressure p at any point is obtained by dividing the total force or thrust exerted on the surface by the area of that surface, *i.e.*,

$$p = \frac{P}{\alpha} \qquad \ldots \quad (80)$$

If the pressure on a surface be *not* uniform, (as on the vertical side of a vessel containing a fluid), so that the force on each equal element of the surface is not the same, the pressure at a point A of the surface is the force which will be exerted on a unit area at A, if on this unit area the pressure be uniform and the same as it is on an indefinitely small area round A.

The dimensions of pressure

$$= \frac{[\text{Force}]}{[\text{Area}]}$$

$$= \frac{[M][L][T]^{-2}}{[L]^2}$$

$$= [M][L]^{-1}[T]^{-2}.$$

The pressure at a point within a fluid is expressed in dynes or in grammes weight per square centimetre, or in pounds weight per square inch.

124. Pressure within a Liquid.—If a liquid mass be at rest, its weight produces internal pressures within its mass and on the sides of the vessel containing it. Suppose the mass of a liquid is divided into a large number of thin horizontal layers. It is obvious that each layer has to support the weight of the layers above it. Thus a layer at a certain depth below the surface of the liquid experiences a downward force greater than that experienced by one which is above it. Hence it

follows that the pressure at a point P is *proportional to its depth.*

Expt. 22. Stretch a piece of India-rubber sheet across the mouth of a thistle funnel T and connect it with a glass tube by means of a length of an India-rubber tubing. A drop W of water, introduced with n the tube G, will act as an index of pressure (Fig. 114). For if the rubber on T is pressed with a finger, the air enclosed in the rubber tube is compressed and the index of water will move forward. the distance moved through being greater as the pressure of the finger is increased. When the pressure is taken off, the index reverts to its old position.

Immerce T to different depths below the surface of water. The motion of the index W will indicate that the pressure on the stretched rubber piece increases continually as the depth is increased.

It is also clear that the pressure at the same point depends upon the *density* of the liquid. Thus if a liquid be twice as dense as water, it will exert at P twice as much pressure.

125. Equality of Pressures in all Directions.— The pressure at a point within a liquid is exerted in *all* directions about the point and is the *same* in value.

If a plane area be rotated within a liquid mass about a point P, each face of the area is acted upon by a thrust perpendicular to the plane. Hence the

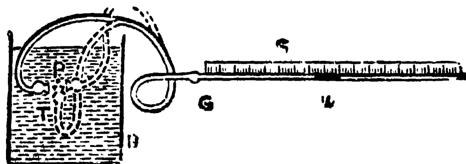


Fig. 114

Equality of pressures on all sides at P

pressure at a point will have different directions according to the orientation of the plane. Thus

water rushes into a boat through a leak in the bottom as well as through a hole in the side.

Again, this pressure is the same in value, whether the surface which experiences the pressure, faces upwards, downwards, horizontally or obliquely.

To show the equality of these pressures, the pressure-gauge of Expt. 22 may be used.

Expt. 23. Take the funnel with its face downward to a depth below the level of water and note the position of the water index. The gauge measures the vertical upward pressure. Now turn the funnel so as to make its mouth vertical, keeping the average depth the same as before (Fig. 114). The gauge will now measure the lateral pressure.

Thus turn the funnel-mouth to different directions keeping it at the same depth. The stationary position of the index shows that the magnitude of the liquid pressure is the same.

126. Vertical Upward Pressure.—Imagine a horizontal lamina drawn in the liquid mass. The weight of the liquid above it is the force acting *downwards* on its upper surface. In order that equilibrium may be obtained, there must be an *upward vertical thrust* across the plane, equal to the weight of the liquid above it. This upward pressure is sometimes termed the **buoyancy** of liquids.

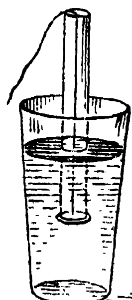


Fig. 115 Upward pressure in a liquid.

Expt. 24. Take a glass tube, one end of which is ground. Hold a thin plate of glass or a metal against the bottom of the tube with a string attached to it (Fig. 115). Lower the whole to some depth into a vessel of water and release the string. The plate does not fall. It is kept in its position by the upward pressure of water.

Pour water (which may be coloured) slowly in the tube. If weight of the plate be small, it will be found to drop when the height of water inside the tube is almost the same as that of the water outside.

This shows also that the upward pressure on the

plate is equal to the downward pressure of a column of water standing on the plate.

127. Lateral Pressure.—The existence of the lateral pressure which a liquid exerts upon the sides of the vessel which contains it, may be understood by supposing that a hole is made on the side of the vessel and that this hole is covered by a plate which exactly fits the hole. The plate will remain at rest only when some external force is applied to it.

When a liquid mass is at rest, the horizontal pressures on the two sides destroy each other, for it is a known fact that a vessel shows no tendency to any horizontal motion by being simply filled with a liquid. Accordingly, if we remove an element of one side of the vessel leaving a hole through which the liquid can flow out, the rest of the pressure against this side will be insufficient to preserve the equilibrium and there will be an excess

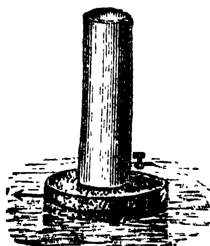


Fig. 116

Backward movement of a discharging vessel

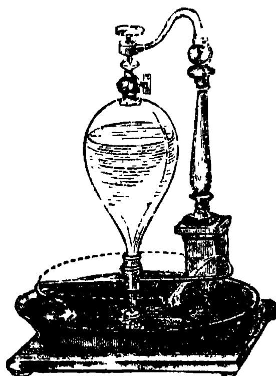


Fig. 117

Hydraulic tourniquet or Barker's mill

of pressure in the opposite direction.

Expt. 25. Fig 116 represents a floating vessel of water having a stop-cock on one side. Fill it with water and open

the stop-cock while the vessel is at rest. Water jets out, and the vessel is seen to move slowly in a direction opposite to that of the issuing stream.

In a *Hydraulic Tourniquet* or Barker's Mill (Fig. 117) the effects due to the same cause are more marked. This consists of a vessel of water free to rotate about a vertical axis. The bottom of the vessel is provided with horizontal discharging arms, all bent in the same direction at the ends. Water emerges by the apertures of the bent tubes, and the mill is seen to rotate rapidly. The unbalanced pressures at the bends of the tubes opposite to the openings cause the apparatus to rotate in a direction opposite to that of the issuing stream of water.

128. Magnitude of the Pressure.—We have defined the pressure at a given point as the force exerted on unit area described about that point. Let us now find an expression for the pressure at a given point in a liquid. Take a small circular area of magnitude A (Fig. 118) surrounding a point P inside the liquid at which the pressure is to be determined. Let the density of the liquid be ρ and the depth of the point P be h. Imagine a vertical column of the liquid whose base is the circular surface A. This liquid column is in equilibrium under the action of three forces : (1) the pressure of the surrounding liquid (2) its own weight acting vertically downwards and (3) the upward thrust acting on the base. The pressure of the surrounding liquid acts horizontally at all points on the vertical sides of the column and has no component in the vertical direction. Hence the two remaining forces must balance so that the

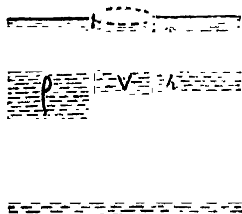


Fig. 118
Pressure in a liquid.

upward thrust on the base is equal in magnitude to the weight of the liquid column. That is,

Upward thrust on A = mass of the liquid column × g = volume of the column × density of the liquid × g

$$= A\,h\rho g$$

$$\therefore \quad \text{Pressure at P} = \frac{A\,h\,\rho\,g}{A}$$

$$= h\,\rho\,g \qquad \ldots \quad (81)$$

From the above relation we find that the pressure at any point in a liquid depends upon

(a) the depth of the point

and (b) the density of the liquid.

Fig. 119 represents a tall vessel with several holes on its side at different depths. When the vessel is filled with water, the water is seen to flow out through the holes with different forces, for the pressures inside the liquid at the levels of holes vary, being greater at a greater depth.

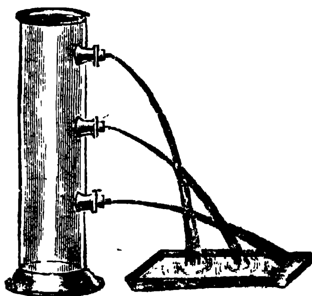


Fig. 119
Pressure at different depths

129. Hydrostatic Paradox.—Since the amount of the total pressure on a horizontal area A at a depth h in a liquid of density ρ is $Ah\rho g$, it follows that the pressure on the bottom of a vessel containing a liquid is not affected by the breadth or narrowness of the upper part of the vessel. Thus if a number

of vessels of different shapes have bases of the same area A, and are filled to the same height h, with the same liquid of density p, the total pressure on the bases should be exactly the same in all the vessels.

This conclusion is sometimes called the *hydrostatic paradox* ; for, at first sight it seems quite impossible that the small quantity of liquid in the vessel C (Fig. 120) can exert the same force on the bottom as a much larger amount of liquid in the vessel A, although having the same base area. But the following experiment due to PASCAL fully verifies the conclusion.

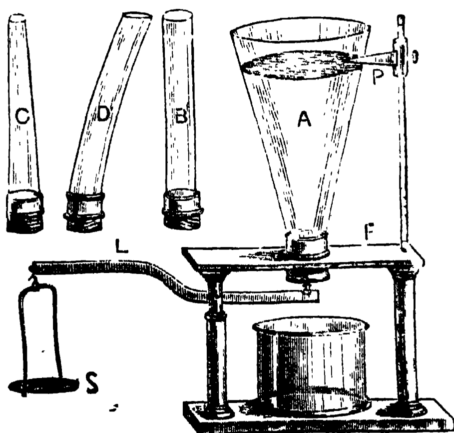


Fig. 120

Pressure on the base of a liquid vessel

Expt. 26. The vessels A, B, C, D in fig. 120 known as *Pascal's vases* are open at the base and have apertures of the same area. They can be screwed on to the platform F. A disc attached to one end of a lever L (or of the beam of a balance) presses against the bottom of any of the vessels when screwed

on to the stand. The other arm of the lever or (balance) carries a scale-pan S on which weights can be placed.

Screw on to F one of the vases. Place a suitable weight on the scale-pan. Pour water into the vase until the disc just begins to give way and allows some water to escape. Note the level of water by means of the adjustable pointer P. Remove the vase and replace it by another, and repeat the same experiment. It will be found that water begins to escape, when it attains the same level as before.

Thus the thrust on the base depends only on the height of the liquid and *not upon* the shape of the vessel or the quantity of liquid contained.

130. Pressure on the Support.—It should be noted, however, that in the last article we examined the thrust on the base of the vessel containing a liquid. But the force on the table on which the vessel stands, does in all cases consist of the weight of the vessel itself, together with the weight of the contained liquid. For obviously the table has to support both these forces. Thus the force on the supporting table depends on the size and shape of the vessel containing the liquid while the pressure at the base of the containing vessel is dependent only on the height of the *free* surface of the liquid above the base.

Suppose that three vessels of the shapes shown in fig. 121 are filled with the same liquid of density

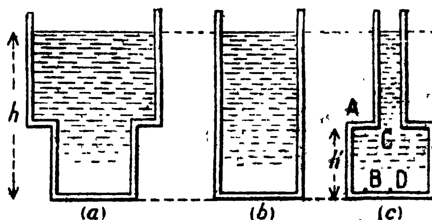


Fig. 121

ρ to a given height h. Then the pressure exerted at

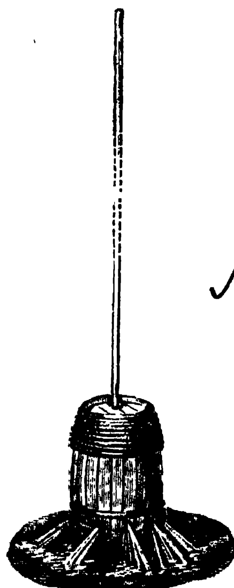
any point of the base of the vessel is in each case equal to $h \rho g$. But if these vessels are supported on a table the force exerted on the table will be greatest in case of the vessel of fig. 121 (a) and least in case of fig. 121 (c)

It may at first sight seem perplexing how the pressure at a point B on the base of the vessel of fig. 121 (c) vertically below the inward projection AC is equal to $h \rho g$. For the beginners may argue that the height of the liquid above B is h' which is much less than h. It should be noted, however, that the pressure at B is equal to the pressure at the point D on the same horizontal level for if it were smaller or greater a horizontal motion of the liquid particles would ensue. But since the liquid is at rest, there is no such motion.

Hence

$$\text{Pressure at B} = \text{pressure at D} = h \rho g$$

It should be emphasized, therefore, that the pressure at any point inside a liquid is dependent on *the depth of the point below the free surface* of the liquid whether it is vertically below the free surface or not.



Hydrostatic Paradox
Fig. 122

The following experiment, first performed by Pascal, may be made to demonstrate the truth of the above statements. A long narrow tube is fixed to the top of a stout wooden cask (Fig. 122) and the

fixed tube and the cask are then filled with water. The pressure at any point on the bottom of the cask is equal to $h \rho g$ and therefore the total pressure, on the bottom is equal to $A h \rho g$, where A is the base area of the cask, h is the height of the surface of water in the tube above the base of the cask and ρ is the density of water. Thus, although the actual weight of the water contained in the tube is very small, its volume being very small, the total pressure at the bottom of the cask is equivalent to the weight of a vertical column of water whose base area is equal to that of the cask and whose height is the height of the liquid surface in the tube above the base of the cask. This may be too great for the cask to bear which ultimately bursts.

But the force exerted on the support on which the cask stands is equal to the weight of the cask and tube together with the total quantity of water contained in the whole system and is evidently much smaller than the total pressure exerted on the bottom of the cask by the superincumbent liquid.

131. Transmission of Fluid Pressure; Pascal's Law.—If the pressure at any point of a fluid is changed, the pressures at all other points are changed also. PASCAL, the great French scientist and mathematician (1623-1662) first asserted in his treatise on 'Equilibrium of Liquids' that *the pressure exerted anywhere on a mass of liquid is trasmitted in all directions so as to act with undiminished force per unit area.*

This transmitted pressure does, as we have seen before, always act perpendicularly to any surface in contact with liquid mass, as otherwise it would have a tangential component, the existence of which is denied by the fundamental property of a fluid at rest (see art. 122).

Imagine a vessel provided with openings of vari-

ous cross-sections *a*, *b*, *c*, *etc.*, closed by a water-tight movable piston (Fig. 123). Suppose the vessel to be filled, with a liquid, and suitable forces be applied to the pistons to keep them in their places. If now an additional pressure *p* per unit area be applied, so that the total force on the piston of area *a* is equal to *pa*, all the other pistons will be forced out. In order to maintain the equilibrium of the liquid mass, it is necessary to apply an additional force to each of the pistons, which is given by the product of the area of the piston and the increased pressure (*p*) per unit area. If the piston are all of the same area, the additional force to maintain each piston in position will be the same.

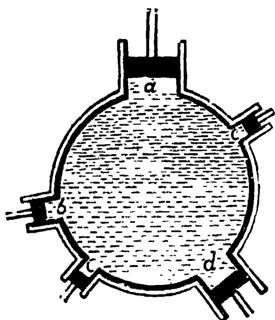


Fig. 123

132. Multiplication of Force by the Transmission of Fluid Pressure.—In fig. 124, *A* and *B* are two cylinders of different sectional areas, say α and β respectively; they communicate with each other through a pipe *C*. They are filled with water. A downward force *w* applied to the piston P_1 in *A* will support a heavy weight *W* acting on the

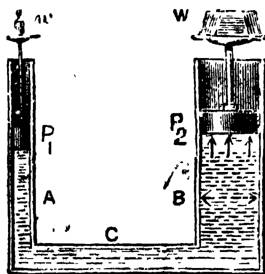


Fig. 124
Multiplication of Pressure

larger piston P_2 in B in order to maintain equilibrium for the thrust per unit area of the piston in $A = w/\alpha$, which is transmitted to each unit area of the piston in B. Hence the total upward thrust on the piston in

the vessel B $= \frac{w}{\alpha}\beta$, which equals W.

$$\text{or} \quad \frac{W}{w} = \frac{\beta}{\alpha} \qquad \ldots \quad (82)$$

Now if the area of B be 50 times that of A, one kilogram applied at A would support 50 kilograms at B. Thus a very small force may be transformed into one of a large magnitude. This principle is utilized in the Hydraulic Press.

In a *Hydrostatic Bellows* designed by PASCAL himself the principle of transmissibility of pressure in a fluid is illustrated. A stout India-rubber bladder or a leather bellows (Fig. 125) is attached to a narrow tube A. The tube is fixed in a vertical position and the bellows B rest on the table. The bellows and a part of the tube are then filled with water. It may be seen that a heavy weight placed on board of the bellows will be supported simply by the weight of small column of water in the tube.

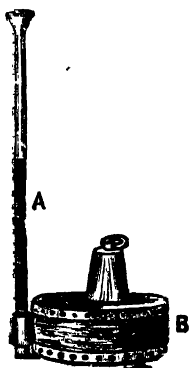


Fig. 125
drostatic Bellows

133. The Hydraulic Press.—The hydraulic press is in common use wherever a great pressure is requir-

ed. Thus it is applied in compressing paper, cloth, cotton etc.; in testing the strength of iron beams, steel plates for armour-clad vessels; in extracting oil from seeds; in making dies, in embossing metal etc.

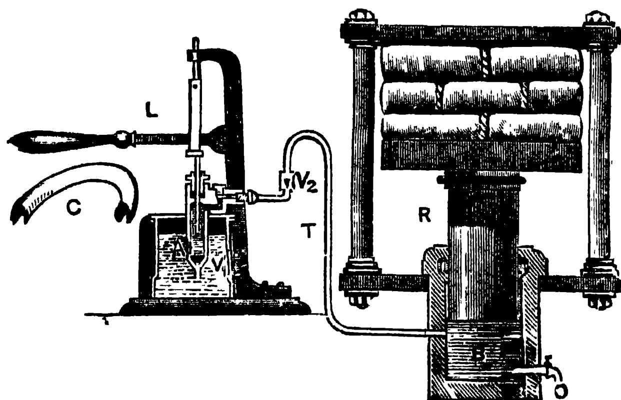


Fig 126
The Hydraulic Press

It consists of two iron cylinders or barrels, A and B (Fig. 126) the section of B being much greater than that of A. There is in each cylinder a solid rod plunger acting as a piston. The piston in A can be moved up and down by means of a lever L, having its fulcrum at its one end. The piston in the cylinder B, which is a large iron ram R, carries a platform on its top, whereon the objects to be pressed are placed. There is a fixed plate above the platform supported on iron pillars. Two valves, one placed in A, say V_1, another in the connecting tube T, say V_2, allow passage to water in one direction only.

As the piston in A is raised by means of the lever L, the pressure in the barrel A is diminished

and the water from the cistern below opens the valve V_1 and enters the cylinder. When the piston comes down, the valve V is closed, the second valve V_2 opens and the water passes into the large cylinder B. The pressure applied on the piston in A is thus transmitted to the second piston and is many times multiplied. The platform rises slowly with a very large pressure. The valve V_2 prevents the water from rushing back from the barrel B.

Let F be the force applied to the lever.

The mechanical advantage $= \dfrac{\text{Power-arm}}{\text{Resistance-arm}} = m$, say.

The downward thrust F_1 on the piston in A $= mF$ which is distributed over an area α of the piston.

Hence the upward thrust F_2 on the larger piston in B of area β is given by Art. 132.

$$F_2 = F_1 \times \frac{\beta}{\alpha} = m.F.\frac{\beta}{\alpha} \quad \ldots (83)$$

Therefore, the mechanical advantage of the whole machine

$$= \frac{mF.\beta/\alpha}{F} = m.\frac{\beta}{\alpha}$$

Thus if the long arm of the lever L is 10 times that of the short arm, and the area of the larger plunger is 50 times that of the smaller one, the force applied is multiplied 10×50 or 500 times.

Though Pascal illustrated the principle of multiplying a force hydrostatically, his apparatus (Fig. 124) was practically useless for many years on account of the difficulty in making the piston in B water-tight. Engineer Bramah was the first to get over this difficulty (1796) by inventing the cupshaped leather collar which prevents water from escaping by the sides of the piston. It consists of a ring of leather, semi-circular in section, which is fitted with its concavity downard round the piston in a hollow in the sides of the cylinder (C in Fig. 126). Water in passing between the piston and the cylinder fills the hollow

under the ring and produces by its pressure on it a packing which becomes the more tight as the pressure on the piston increases.

It is evident that the decrease in the volume of water in A is equal to the increase of water in B.

$$\text{Hence} \quad x.\alpha = y.\beta$$

if x is the distance through which the small plunger of area α has to move down to raise the big plunger of area β through a distance y

$$\text{Or } \frac{x}{y} = \frac{\beta}{\alpha} = \frac{F_2}{F_1} \quad \text{(Eqn. 84)}$$

$$\therefore \quad F_1 x = F_2 y$$

As *work = force × distance*, the above equation indicates that the work done by the larger piston is the same as that done on the smaller one. Thus, in the example given above, the smaller piston has to come down through 60 times the distance through which the larger piston is raised. Again the end of the lever-rod where the power is applied, has to be pushed down through 600 times the same distance.

Thus the Principle of Work or the Conservation of Energy holds true in this case.

Exercise XIII

1. Describe experiments to show that water exerts pressure in all directions. [C. U.—1911 ; '14 ; '27]

2. A tin vessel provided with a tap at the side near the bottom is filled with water, and made to stand upright on a thick plate of cork. Explain what will happen when the tap is opened. [C. U.—1914]

3. A plate 10 mm. square is placed horizontally a metre below the surface of water, when the height of mercury barometer is 73 cm. What will be the total pressure on the plate? (Density of mercury —13·6 gms, per cc.) [C. U.—1911]

4. Define intensity of pressure at a point P in a fluid.

Prove that the difference of pressure between the surface of a liquid and a point in the liquid z cm. below the surface, is given by $p = gdz$, where d is the density of the liquid and g, the acceleration due to gravity. [C. U.—1910]

5. Find the pressure due to a column of mercury 50 cms. high. Does the pressure vary with the diameter of the tube in which the mercury is made to stand ?

6. A cube, the edge of which is 10 cms. is suspended in water with its sides vertical and its upper surface at a depth of 10 cms. below the surface. Find the pressure on each of its faces.

7. State Pascal's law as to the transmission of pressure in a liquid.

Describe the principle and action of a Hydraulic Press, giving a sectional diagram. [C. U.—1922 ; '29]

8. Describe experiments by which you will measure the pressure of the gas and water supplied in the laboratory at the respective taps on your working bench. [Pat. U.—1919]

9. The density of sea-water is 1·025. Find the pressure at the depth of 10 ft. below the surface in pounds per square foot ; given that one cubic foot of water weighs 62·5 lb. [C. U.—1927]

10. At what depth below the surface of water will the pressure be equal to two atmospheres, if the atmospheric pressure is 1 megadyne (10^6 dyne) per sq. cm. ?
($g=981$ cm./sec.^{2}.)

CHAPTER XVI

EQUILIBRIUM OF LIQUIDS.

134. The Atmospheric Pressure.—It is known to all of us that the earth is surrounded by an atmosphere of air which is a mixture of nitrogen, oxygen and other gases. We are literally immersed in an ocean of air, just as a fish lives in an ocean of water. Air, being a fluid, thus exerts pressure from all directions on our bodies and on all objects on earth with which it is in contact. As we rise up above the surface of the earth the amount of air above us gradually diminishes and so the pressure also diminishes, for this is equal to $h \rho g$, where h is the height of the surface of air above the point in question, ρ is the density of air and g the acceleration due to gravity. The pressure which the air around us exerts at a point is called **atmospheric pressure**. At the surface of the earth the magnitude of this pressure is about 10^6 dynes per sq. cm.

A more detailed discussion about the atmosphere and the pressure exerted by it will be given later on.

135. Conditions of Equilibrium of Liquids.—We have already seen that a liquid mass cannot be at rest in any vessel unless the pressures exerted on a liquid molecule at any point act from all directions and are equal in value. The truth of the above condition is self-evident, for if the forces exerted on any particle in two contrary directions were not equal, the particle would move in the direction of the greater force, and there would be no equilibrium.

Further, when a liquid is at rest under gravity, as is generally the case, its free surface must be horizontal. As otherwise, some parts as at A would be higher than others as at C (Fig. 127). The pressures at B and D of a horizontal layer BD, taken within the liquid mass will be unequal. For,

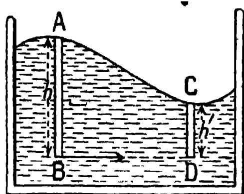


Fig. 127
The free surface of a liquid at rest

Press. at $B = \pi + g\rho h$
and Press. at $D = \pi + g\rho h'$
where π is the atmospheric pressure, $h = AB$ and $h' = CD$.

AB being greater than CD, the pressure at B is greater than that at D. Hence the liquid particles which are mobile, would move away from the places of higher pressure to those of lower ones ; in other words, the mass of the liquid cannot be at rest.

The state of equilibrium will be attained only when the pressures at B and D are equal, *i.e.*, when the liquid surface is horizontal.

It is to be noted from the above that the pressures at all points in the same horizontal plane are equal. This may also be seen in another way:

Take two points A and B in the same horizontal plane (Fig. 128) within the liquid. Join AB and consider a very thin cylinder having AB as its axis.

This cylinder is in equilibrium under the action of the following forces :—

(i) its weight acting in a vertical direction and therefore perpendicular to AB.

(ii) The thrusts on the curved surface, everywhere perpendicular to the surface and therefore perpendicular to AB. and

(iii) The thrusts on the ends at A and B, these being the only forces in the direction of the axis AB.

The forces under (i) and (ii) being perpendicular to AB have no component in the direction of AB.

Hence, the forces under (iii) must balance, *i.e.*,

the thrust on the end A = the thrust on the end B.

And since the areas of the ends A and B are very small and equal,

pressure at A = pressure at B.
For equilibrium,

the thrust on the end A = the thrust on the end B.

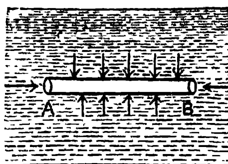


Fig. 128
Pressure in a horizontal plane

If the ends of the cylinder are very small, we have pressure at A = pressure at B.

136. Equilibrium of a Liquid in Communicating Vessels.—When a liquid is contained in several vessels communicating with each other and is at rest, it stands equally high in all the vessels, so that

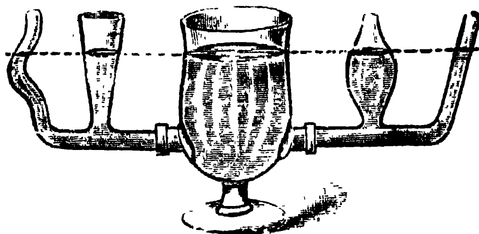


Fig. 129
A liquid in communicating vessels

the free surfaces of the liquid in all the vessels lie in the same horizontal plane. Thus in a tea-pot the water stands at the same height in the spout as in the vessel itself. This fact is commonly expressed by saying that LIQUIDS FIND THEIR OWN LEVEL. This is, of course, a consequence of the fact that the pressures in a horizontal layer in the liquid at rest are the same.

Expt. 27. In fig. 129 the apparatus consists of a series

of vessels of different shapes and capacities, connected together by a common tube. Pour water into one of these vessels. The water-level is seen to rise in all the vessels, and will stop at the same height in each.

For equilibrium, the pressures at points in the same horizontal plane within the liquid must be the same: again, because the pressure at any point depends upon the height only of the liquid above it and not on the shape of the vessel that contains it (art. 128), the level of water in all the vessels is in the same horizontal plane.

137. Equilibrium of Several Liquids in the Same Vessel.—When several liquids of different densities, which do not mix, are placed in the same vessel, they will arrange in the order of their densities decreasing from the bottom upwards.

Expt. 28. Pour into a long narrow bottle small quantities of mercury, spirit coloured red, water saturated with potassium carbonate to prevent its mixing with alcohol, and kerosene oil. Shake the bottle and then keep it aside on the table.

The liquids are first seen to mix but gradually they will separate. Mercury, the densest liquid sinks to the bottom; then stand water, alcohol and oil [Fig. 180].

For the some reason, the fresh water at the mouth

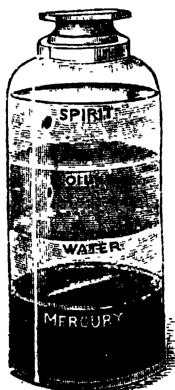


Fig. 180
A phial of four liquids

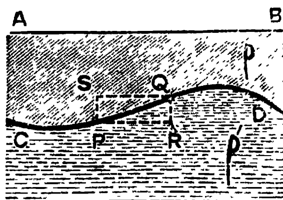


Fig. 181
The common surface of two liquids which do not mix

of a river floats for a long time on the denser salt-

water of the sea. Cream, which is lighter than the milk, rises to the surface.

Secondly, the common surface of any two liquids, which do not mix, is a *horizontal plane.* The statement may be verified by observing the common surface of any two liquids in the bottle (Fig. 130), when the liquids are at rest.

Theoretically, the statement may be proved in the following way :

Let P and Q [Fig. 131] be any two points on the common surface CD of two liquids of densities ρ and ρ'. Let PR and QS be the horizontal planes through P and Q.

$$\text{Pressure at P} - \text{pressure at S} = g\ \rho\ h$$

$$\text{Also pressure at R} - \text{pressure at Q} = g\ \rho'\ h$$

where h=perpendicular distance between the horizontal planes PR and QS.

$$\text{But pressure at P} = \text{pressure at R}$$

$$\text{and pressure at S} = \text{pressure at Q}$$

$$\therefore g\rho h = g\ \rho'\ h$$

$$\rho h = \rho' h$$

i.e., ρ and ρ' must be equal or if they are unequal, h must vanish. *i.e.*, P and Q must lie in the same horizontal plane.

138. Equilibrium of Two Liquids in Two Communicating Vessels.—If two liquids of different densities, which do not mix, be contained in two communicating vessels, their levels are no longer in the same horizontal plane, the level of the lighter liquid being higher.

There will be equilibrium only when the heights of the two liquid columns in the two vessels above their common surface of contact are inversely as their densities.

Let us suppose that two liquids of densities ρ_1 and ρ_2 occupy the portions CED and AD respectively

of an U-shaped tube as shown in fig. 132, their common surface of separation being D. Draw a horizontal plane through D cutting the other arm in E.

Then evidently the pressure at D is equal to the pressure at E. Let $CE = h_1$ and $AD = h_2$

Then pressure at $E = \pi + \rho_1 g h_1$

and pressure at $D = \pi + \rho_2 g h_2$

$$\therefore \quad \pi + \rho_1 g h_1 = \pi + \rho_2 g h_2,$$

where π is the atmospheric pressure.

$$\text{Or} \quad \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \qquad \dots \qquad (86)$$

Hence in order that two liquids in communicating vessels may be in equilibrium, their heights above their common surface shall vary *inversely* as their densities.

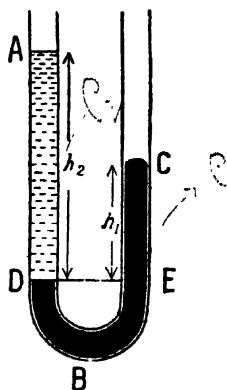


Fig. 132
Equilibrium of two liquids

The relation is made use of in comparing the densities of two liquids (Art. 151).

It is to be noted that as the column of liquid supported in each vessel depends on the pressure per unit area, the cross-sections of the tubes do not at all enter into consideration.

139. The Water-level.—This instrument embodies a simple application of the principle that the levels of a liquid in two connecting tubes is in the same horizontal plane. It consists of a metal tube (fig. 133) bent at right angles at both ends, in which are fitted two small glass tubes. The apparatus is placed on a tripod stand which allows the instrument

to be turned to any direction. It is placed in an approximately horizontal position, and water generally coloured a little, is poured into the tube until it rises into the glass limbs. When the liquid comes to rest, the surfaces of water in the two legs are in the same horizontal plane.

The water-level is used in the operation called *levelling*, the object of which is to ascertain the difference between the levels or the vertical heights of two

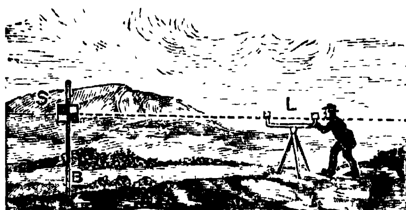


Fig. 133
Using a water-level

given points. At each of the points a levelling-staff is fixed. This is an upright graduated rod on which slides a small square tin plate, whose centre serves as a mark for the observer. The observer looks across the surface of water in the two tubes of the level, and directs his assistant to raise or lower the plate until its centre is in his line of sight (Fig. 133). The assistant goes next to the other point and repeats the process. The difference of the levels required is given by the difference in the positions of the slide on the staff.

For more accurate work, a telescope with an attached spirit-level is used. The observer reads the graduation on the levelling staff through the telescope which is kept horizontal by means of a spirit-level.

140. The Spirit-level.—It consists of a glass tube very slightly curved and contains spirit which nearly fills the tube, leaving room for a bubble of air which occupies a position at the highest part of the tube. It is fixed in a brass mounting (Fig. 134). When the case rests on a horizontal surface, the bubble is exactly between two lines marked on the glass by the maker. But if the surface be inclined to the horizon, the bubble will always stand nearest to the higher end of the tube. By reversing the spirit level, the bubble will change its place in the tube. The instrument, therefore, furnishes a means of testing the horizontality of the surface on which the level rests. The spirit-level is more delicate than the water-level.

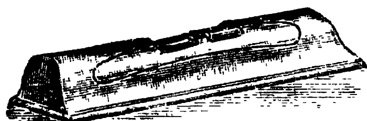


Fig. 134
A straight Spirit Level

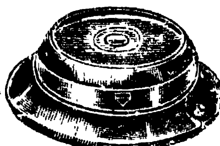


Fig. 135
A circular Level

To examine whether a plane surface is horizontal or not, it is necessary to test by means of a spirit-level whether any direction taken in the plane is horizontal or not.

Fig. 135 represents the circular form of a spirit-level often used in the laboratory. The spirit is in a small circular vessel with a glass top which forms part of a sphere, and the under surface of which is concave. A bubble is allowed to remain inside the vessel and a circle is etched on the glass top, so that if the circle marked by the bubble is concentric to this, the base of the vessel is horizontal.

141. Water supply of Towns.—The property of a liquid 'to find its own level' is utilized in the

water-supply of towns. A reservoir is constructed on some elevation which is higher than any part of the town to be supplied. Water is forced up by a pump into the reservoir, which is fed by a river, a lake or a spring. Main pipes starting from the reservoir, are laid along the principal roads, and smaller service-pipes are branched off from these mains to go to the houses to be supplied. Water will find its way from the reservoir to the houses through the pipes which may rise or fall, in whatever manner is convenient, provided that no part of these is higher than the surface of water in the reservoir.

142. Artesian Wells. – The action of the Artesian wells depends on the same tendency of water to find its own level. Fig. 186 represents a section of what the geologists call an Artesian basin. The layer K in the earth's crust is composed of some porous materials, such as sand, gravel or chalk, through which water can percolate. The strata B and C above and below K respectively are clay, slate or some other material, impervious to water. The rain-water falling on that part of the stratum K where it comes to the surface, called the *outcrop*, will collect in K. If now a boring is made through the layer B, the water gushes forth to a height that depends on the difference between the levels at the outcrop and at the part where the boring is made.

Many Artesian wells exist in the United States, Algeria, Australia, Germany etc. A number of Artesian wells has been bored in the desert of Sahara and an abundant supply of water

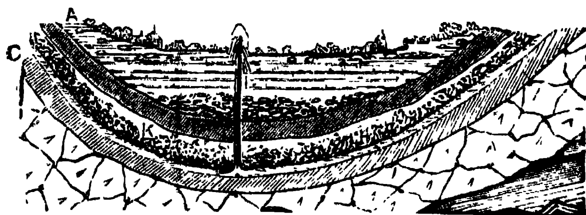


Fig. 186
An Artesian well

is found at a depth of 200 ft. The water which feeds the

Artesian wells often comes from a very great distance. The deepest Artesian well in existence is near Berlin. It is 4194 ft. deep.

Tube-wells.—A **tube-well** is a device by means of which we can pump out water from inside the earth. This underground water is found at levels which are usually much less deep than in the case of Artesian wells but are fed by sources which are lower than the surface of the earth at the place where the tube well is bored. Consequently, the water has not enough pressure to come out of the earth's surface of itself so that mechanical effort is to be made to pump it up.

Exercise XIV

1. Explain what is meant by saying that *water finds its own level.* Give any practical illustration of this.

2. Describe a spirit-level and how it is used to level a plane surface.

CHAPTER XVII

RESSURE ON BODIES IMMERSED IN LIQUIDS

143. Thrust on a Body immersed in a Liquid —It has been shown in art. 127 that a body immersed in a liquid experiences an upward force. A piece of cork, plunged in water and then let go, is pressed up towards the surface and floats. It is easy to lift a tub of water within water, but as soon as it leaves the water, its full weight is to be supported. A person bathing in a tank can support his whole weight by pressing lightly against the ground under water with his fingers, or may tread upon sharp stones without any injury, as he is buoyed up by the water.

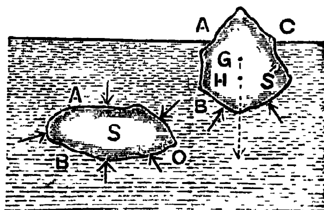


Fig 137.
Thrust on an immersed body

Let S (Fig. 137) be a body immersed in a liquid at rest under the action of gravity and held in position, if neccssary. The solid is subjected to the pressure exerted by the surrounding liquid. Imagine the body S to be removed and the space ABC it occupied, be filled with extra liquid, the rest of the liquid being undisturbed. The pressure on each element of the surface ABC of this additional

liquid has the same value as when the solid was immersed and hence the resultant thrust on the solid is the same as that on the liquid which has replaced it. Now the latter is in equilibrium under its own weight W acting vertically downwards through its centre of gravity, and the resultant of the pressures due to the surrounding liquid acting on its surface. Hence *the resultant thrust must be equal to the weight of the liquid of the same volume, and must act vertically upwards through the centre of gravity of this portion of the liquid.*

If the solid be not completely immersed, the same conclusion is also arrived at by supposing the immersed portion of the solid to be replaced by the liquid.

The upward resultant thrust on a solid immersed in a liquid, wholly or partially, is called the **buoyancy** of the body ; the centre of gravity of the liquid displaced is the **centre of buoyancy**.

The centre of buoyancy of a body does not in general coincide with its centre of gravity. The two points will coincide, only when the body immersed is homogeneous. It should be noted, however, that they are always on same vertical line.

The resultant thrust in a liquid does evidently depend on the density of the surrounding liquid and the volume of the immersed body, and is not affected in any way by the nature of the material of the body immersed.

A body experiences no buoyancy, however, unless the liquid acts from below the body. This explains why boats which have settled down on a muddy bank at low-tide do not sometimes rise with the rise of tide.

144. Principle of Archimedes.—The deduction arrived at in the last article is known as the Prin-

ciple of Archimedes, the celebrated Sicilian geometrician of antiquity. The principle enunciates that "*A body immersed in a liquid seems to lose a part of its weight, which is equal to the weight of the displaced liquid.*"

The displaced liquid has evidently the same volume as that of the body immersed; and the loss in weight of the immersed body is caused by the upward thrust equal to the weight of liquid of the same volume as that of the body.

Archimedes' Principle may be verified experimentally by means of a hydrostatic balance or a spring-balance. A **hydrostatic balance** is simply an ordinary balance by which the weight of a body immersed in a liquid is conveniently obtained. In some forms, the balance has one of its pans suspended by a shorter frame or chains than the other pan. A hook is attached to this pan, from which the body to be weighed is hung by a string. In other forms, as in fig. 138 a wooden stool or bridge C is placed on the floor of the balance-case over one of the scale-pans (say, the left-hand one) which can swing freely below it, the supports of the scale-pan passing on either side of the bridge. A beaker may be placed on the bridge, and a body may be hung immersed in a liquid in the beaker without interfering with the free movement of the pan, when the beam of the balance oscillates.

Expt. 29. In fig. 138, A is a solid metal cylinder with a hook attached to its upper end. B is a hollow cylinder closed at the bottom. A just fits in B, so that the internal volume of the cylinder B is equal to the volume of A.

Suspend A below B, and the whole from a hook attached to the knife-edge on the left-hand arm of a balance, so that A hangs inside an empty beaker D placed on the bridge. Counterpoise the whole by placing weights on the other scale-pan.

Fill the beaker with water. The upward thrust on A disturbs the balance, and the arm carrying A and B rises up.

Drop water from a pipette into the hollow cylinder B. The balance will be restored when B is full.

Hence the upward thrust on A in water is exactly balanced by the weight of water which fills B, and the volume of water in B is just the same as that of A ; therefore, the upward thrust on A equals the weight of the water displaced by it.

The experiment may be performed with any other liquid too. The liquid in the beaker D and that which fills up B being, of course, the same.

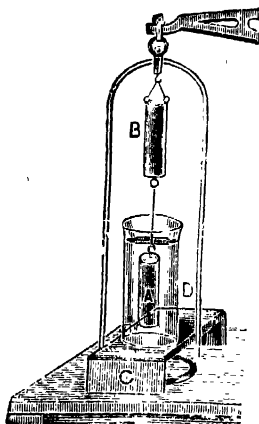


Fig. 138
Verification of
Archimedes' Principle

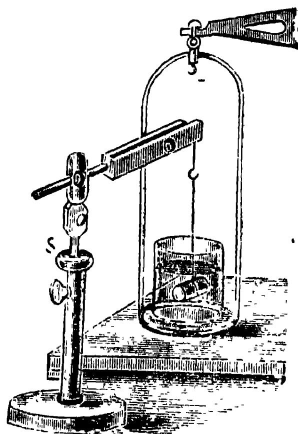


Fig. 139
Thrust of an immersed body
on the bottom of a vessel

The experiment can be more quickly performed by means of a spring-balance

The principle can also be verified by taking an ordinary sinker instead of the cylinders A and B.

Expt. 30. Get an ordinary glass stopper Find its wight W in the usual way. Hang it from the suspension hook of the scale-pan by a fine wire and weigh it as it hangs fully immersed in the water contained in the beaker D of the last experiment.

Record the apparent weight W′, thus observed, of the body in the water. The difference between W and W′ should be

equal to the weight of the water displaced by the body. Now find the volume of the body by the method of displacement of water as explained in art. 20. From the volume in c.c. of the displaced water, its weight in grams is known, for the weight of 1 c.c. of water is approximately 1 gm. It would be found that the weight last obtained agrees very closely with the *apparent loss* in weight of the body in water.

It should be noted that the loss in weight of a body immersed in a liquid is only an *apparent* one, for really a vessel of water and a body, say the solid cylinder A in expt. 29, placed together on the scale-pan of a balance would weigh the same, whether A is outside the vessel or inside it. Though the solid loses a part of its weight while in water, the total weight on the pan remains the same.

Expt. 31. Place a beaker containing water on the scale-pan of a balance and counterpoise it. Thrust your finger in the beaker ; the pan carrying the beaker will go down.

Take a body K whose volume V c.c. is known by the method of deplacement of water. Suspend K in the water from a fixed outside support (fig. 139). The pan carrying the beaker is again depressed. Place weights on the other pan to restore equilibrium. The weight will be found to be V gms ; but this again is the weight of the water displaced by the solid. The immersed body thus seems to add a further weight on the balance-pan equal to the weight of the displaced liquid.

The Explanation of the above may be given in the following way :—The solid when immersed in water displaces a quantity of the water of its own volume, and is thereby acted upon by an upward pressure equal to the weight of the water displaced by it. But as the action and the reaction are equal and opposite (*Newton's Third Law of Motion*), the body exerts, by way of reaction, a downward force of the same value on the water in the beaker. Hence the supporting pan of the balance is affected by this additional weight which is equal to the weight of the water displaced by the body immersed in it.

The story of Hiero's Crown in connection with the remarkable discovery of Archimedes concerning the weight of bodies immersed in water has often been told. Archimedes (B. C. 287-

212) was born at Syracuse in Sicily and lived about the same time as Euclid. 'Hiero', the king had given to a goldsmith a quantity of gold which was to be fashioned into a crown. When his work was completed, the king found that its weight corresponded with that of the metal which had been delivered to the goldsmith ; but he suspected that some of the precious metal had been kept back, and its weight made up by baser metals alloyed with the gold in the crown. He sent the crown to Archimedes to pronounce on the true state of the case. How to do this was for sometime a puzzle to the philosopher, but while thinking over the matter, a slight incident suggested the solution of the problem. He was one day entering his bath, which happened to be quite filled, and noticing that the water overflowed its edge in proportion as he immersed his body in the liquid, it struck him that the quantity of water which thus ran out constituted an exact measure of the bulk of the immersed body which displaced it. He immediately perceived that if the crown were of pure gold, it would, when immersed in a vessel quite full of water, cause the same quantity of the liquid to run over the brim as would a lump of gold of the same weight as the crown ; whereas the latter, if alloyed with silver or bronze, would displace more water than the lump of gold. When this idea flashed upon the bather's mind, he was so overjoyed at this discovery that he leapt from the bath, and ran out, unclad as he was, crying "Eureka ! Eureka ! I have found it out ! I have found it out." Thus led to make experiments on the weights of bodies in air and in water Archimedes came to discover his well-known Principle.

145. Determination of the Volume of a Solid by Archimedes' Principle.—Archimedes' principle provides us with the means of finding the volume of a body that sinks in water. Let

Weight of the body in air = W gm.
Do Do in water = W′ gm.
Then the upward thrust = (W − W′) gm.

This, by Archimedes' Principle, equals the weight of a mass of water equal in volume to the body.

∴ the volume of the body = (W − W′) c.c.
since 1 c.c. of water weighs 1 gm.

Now the density of a body is (see §114.) its mass per unit volume.

$$\text{Hence density of the body} = \frac{W}{W - W'} \text{ gm/c.c.}$$

If the weights are taken in pounds, it must be remembered that 1 cu. ft. of water weighs 62·5 lbs. Hence the volume of 1 lb. of water is 1/62·5 cu. ft. and the volume of (W − W′) pounds is

$$\frac{W - W'}{62·5} \text{ cu. ft.}$$

146. Equilibrium of Immersed and Floating Bodies.—Consider a solid of weight W to be completely immersed in water. Let w be the weight of the liquid displaced. The solid is under the action of two forces, *viz.*,

(1) its own weight W acting downwards through its centre of gravity and

(2) the resultant upward pressure w acting at the centre of buoyancy

Three cases are now possible :

(1) If $W > w$, the solid sinks in water : it is denser than water. Thus a stone, a lump of iron, etc., will sink in water. If it be suspended by a string, the tension T along it must act upwards and is given by

$$T = W - w$$

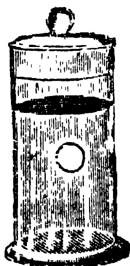


Fig. 140

The body will, therefore, appear lighter within water then in the air.

(2) If $W = w$, the solid will float wholly submerged anywhere in the liquid, and will have no tendency either to ascend or to descend.

Expt. 32. Take a small jar or a bottle. Fill it two-thirds with a mixture of equal quantities of water and alcohol. Drop a small quantity of olive oil in the mixture by means of a pipette. The oil collects into a globular shape due to cohesion and should float in the mixture (fig. 140) somewhere within it. If it falls to the bottom, there is too much alcohol in the

mixture; if it rises to the top, there is too much water. So this can easily be put right.

(3) If $W < w$, the solid floats partly submerged in water. Cork, wood, wax, etc., float on water. As the upward force due to the displaced liquid is greater than the weight of the immersed body, the body is pushed up towards the surface. From the moment it emerges out of the liquid, the weight of the liquid displaced gradually diminishes until it becomes equal to the weight of the solid, when the solid is pushed up no further. It is then in equilibrium under the action of the two equal and opposite forces W and w. In practice, the body may execute a few oscillations at the surface of the liquid before it finally comes to be in equilibrium.

Hence *the conditions of equilibrium of a floating body* are :—

(i) A floating body must displace its own weight of the liquid in which it floats. According to Archimedes' Principle a body that floats has lost its whole weight.

(ii) The centre of gravity of the body and that of the liquid displaced are in the same vertical line, for the lines of action of the two opposing forces W and w acting on a body in equilibrium must be the same.

Thus a piece of wood weighing 200 gms. must displace 200 gms in weight and hence 200 c.c. in volume, of water. It would also displace 200 gms. of any other liquid in which it may be allowed to float. If the density of wood be 0·4, its volume is M/ρ or 200/0·4 or 500 c.c. Thus the piece floats in water with $\frac{2}{5}$ ths of its volume immersed.

When water freezes into ice, it expands in volume, and its density is a little less than 1. A

piece of ice put in a tumbler of water will float in water and will show its top only above the level of water. In sea-water, the density of which is a little higher than 1 (about 1·03), an ice-berg floats out of water with about $\frac{9}{10}$ths of its volume under water.

A body which floats in one liquid, may sink in another. This happens when the body is lighter than one liquid but denser than the other. Thus wax floats on water, but sinks in ether. A lump of iron floats in mercury but not in water. Similarly, drops of oil float in water, sink in alcohol and swim in a suitable mixture of both. An egg will sink if placed in fresh water and will float if placed in a strong solution of common salt in water. For a similar reason a heavily loaded ship is partly unloaded before it enters a river for river-water is less dense than sea water.

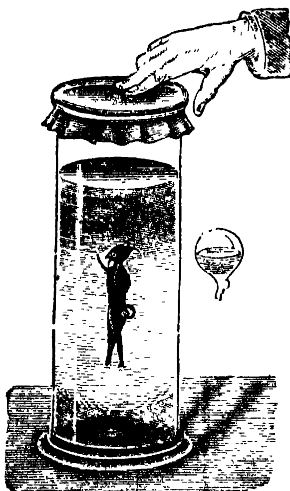


Fig. 141

The Cartesian Diver

The different cases of suspension, immersion and floating that can present themselves when a body is immersed, can be shown by means of a well-known hydrostatic toy, the **cartesian diver**, described even in old treatises on Physics. The diver consists of a small hollow glass ball having an opening in its lower

side, by which water can enter or escape. A little porcelain figure is attached to the ball as a counterpoise. Sometimes the figure has no ball above it, but is itself hollow in the upper part of its body and is provided with a tubular tail open at the end and communicating with the body. The figure is of such a mass that the whole just floats in water with some air in the ball or in the hollow body.

Expt. 33.—Take a jar and nearly fill it with water. Let the diver float in this water. Close the top of the jar with a piece of India-rubber (fig. 141).

Now apply pressure with the hand on the rubber. The diver will sink. On releasing the pressure of the hand, the diver may be made to remain stationary within the water.

What happens is this:—On pressing the India-rubber down, the air above the water in the jar is compressed. The pressure is transmitted through water to the air in the bulb, and compresses it. More water enters the ball and thereby causes the whole toy to become heavier than the weight of the water displaced by it. Hence the diver descends. On removal of the hand, the air in the ball expands, and expels the excess of water which entered it. The figure becomes lighter and ascends. It must be observed, however, that as the diver continues to descend, more and more water enters the ball owing to the increase of the hydrostatic pressure, so that if the depth of the water exceeds a certain limit, the air in the ball cannot expand sufficiently to allow the diver to rise again, even when the air-pressure on the surface is relieved.

Most fishes have an air-bladder, called the *swimming bladder*, below the spine. By compressing or dilating this at pleasure the fish can either rise or sink in water.

The modern **sub-marine** is essentially nothing but a huge Cartesian Diver. When it floats on the surface of the sea like an ordinary ship, it has the greater part of its bulk immersed. It is provided with large **ballast tanks** T (Fig. 142) in the bow, the middle and the stern of the ship. Water may be admitted into the tanks or discharged out of these by means of powerful pumps that are worked by compressed air. When the tanks are full of water, the

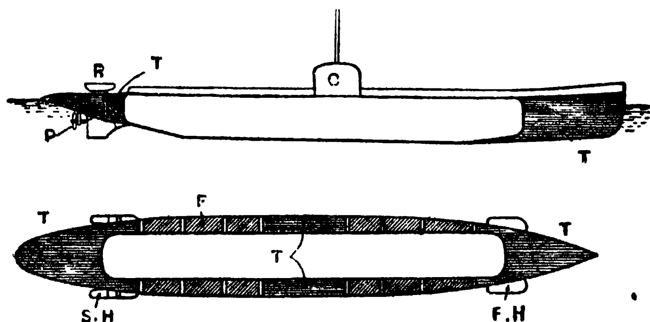


Fig. 142—The Principle of a Sub-marine

ship becomes slightly heavier than the water displaced and hence it sinks. In order to rise up, the water is expelled from the tanks. The operation of filling or emptying the tanks may be done in less than a minute. Besides the vertical **steering rudder** R, to direct the ship in its forward motion, it is also provided with horizontal rudders F.H. and S.H. called the *hydroplanes* in the front and the stern, fitted on both sides of the vessel. By means of engines the hydroplanes can be slightly tilted to regulate the depth of submergence, as desired. To C, the **conning tower** of the ship is fitted the **periscope** P, the top of which remains above the

water surface even when the vessel is submerged; it is used for the purpose of observing what is taking place above the surface.

A body though made of a material denser than a liquid may, however, float on its surface. For this purpose, it must be given a hollow or a concave shape, so that if fully immersed, it can displace a volume of liquid, the weight of which is greater than that of its own. Thus a porcelain saucer, a boat, a ship, etc., float freely on water. Bodies denser than water can also be made to float on water by attaching lighter bodies to them. This principle is applid to the use of life-buoys and life-belts.

Fig. 143 represents a floating dock. When water is allowed to fill the chambers a, the dock sinks until the water-line is at AA′. The ship is then floated into the dock. When it is in place, the water is pumped out of the chambers until the water-line as low as BB′. Workmen can then get access to all parts of the bottom of the ship.

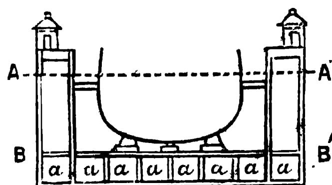


Fig 143.—Floating Dock

147. Swimming—The human body is lighter on the whole than an equal volume of water, the averge ratio being 0·934 : 1. Hence it floats in water. The head of a man is, however, heavier than water and consequently it tends to sink causing thereby a difficulty in breathing. Hence swimming is an art that requires to be learnt, in which practice is to be acquired in keeping the head above the water surface by the muscular action. Air-bladders or cork girdles, known as *life-belts*, are used by persons who are learning to

swim (Fig. 144), as then without any considerable increase in weight, more water is displaced and an



Fig.—144.—Swimming.

increased buoyancy is secured. In quadrupeds, on the other hand, the posterior parts of the body are heavier. Hence they feel no difficulty in keeping their heads above the water and thus swim naturally. Several birds such as ducks, swans, etc., seem to swim almost on the water surface. This is due to the fact that the thick layers of light down covering the lower part of the body act as a hollow impervious coating, so that even with a very small immersion they are able to displace water of weight equal to that of the whole body.

Exercise XV

1. State Archimedes' Principle and explain how it will enable you to identify a given piece of pure metal.

2. Describe a hydrostatic balance, and explain clearly the principle on which the working of the instrument is based.

Given a body A which weighs 7·55 gms. in air, 5·17 gms. in water and 6·35 gms. in another liquid B; calculate from these data the density of the body A and that of the liquid B.
[C. U.—1932]

3. State Archimedes' Principle. How would you demonstrate its truth ?

A body weighs 62 grammes in vacuo and 42 grammes in water ; find its volume. [C. U.—1919 ; '20]

4. How would you show experimentally that the resultant vertical thrust on a body immersed in a heavy liquid is equal to the weight of the liquid displaced ? [C. U.—1924]

5. Under what condition do bodies float or sink in a liquid ?

A piece of iron weighing 272 gms. floats in mercury of density 13·6, with $\frac{5}{9}$ths of its volume immersed. Determine the volume and density of the iron. [C. U.—1930]

6. Explain why an iron ship floats on water.

A hollow spherical ball has an internal diameter of 10 cm. and an external diameter of 12 cm. It is found just to float in water. Find the density of the material of the ball. (The volume of a sphere varies as the cube of the diameter). [C. U.—1928]

7. If you were given a piece of wood cut in the form of a cube, how would you very roughly determine its specific gravity without using a balance ? [C. U.—1911 ; '23]

8. A cubical block of wood of sp. gr. 0·7 floats in water, just completely immersed, when a body of unknown weight is placed on it. Find the weight of this body, if the volume of the block of wood is 100 c c [C. U.—1913]

9. How would you find the specific gravity and the volume of a given solid ?

If the sp. gr. of a metal is 19, what will be the weight in water of 20 c.c. of the substance ? [C. U.—1917]

10. A lump of metal is known to consist of silver and gold. The lump weighs 20 gms. in air and 18·7 gms. in water. How much gold is there in the lump ? (Sp. gr. of gold = 19·3 ; of silver = 10·5). [C. U.—1936]

11. Describe the Cartesian diver and explain how it acts.

Do you know of any modern appliance which is based on this principle ?

A solid body floating in water has one-sixth of its volume above the surface. What fraction of its volume will project if it floats in a liquid of specific gravity 1·2 ? [C. U. 1938]

12. A cylinder of wood (sp. gr. 0·25) has another cylinder

of metal (sp. gr. 8·0) attached to one end. The cylinders are 2 in. in diameter, they have the same axis, and are respectively 20 in. and 1 in. long. If the whole is placed in water, find how much of it will be above the surface. [C. U.—1935]

N. B.—Examples 7-12 should be worked out after going through the next chapter.

CHAPTER XVIII

SPECIFIC GRAVITY.

148. Specific Gravity.—In art. 115 we stated that the heaviness of a substance is indicated by its density, i.e., the mass per unit volume. There is another quantity which expresses the relative heaviness of different substances more clearly. This is the **specific gravity** of the substance which is defined as *the ratio of the weight of a certain volume of the substance to that of an equal volume of some standard substance.*

Thus if W = weight of a given substance
W' = weight of an equal volume of a standard substance
S = sp. gr. of the substance

$$\text{Then} \quad S = \frac{W}{W'} \qquad \ldots \qquad \ldots \qquad (87)$$

When determining the specific gravity of *solids* and *liquids*, the standard substance usually taken is distilled water at a temperature of 4°C. Water is suitable for this purpose, for it can be readily obtained everywhere in a pure condition. As, however, the density of water varies with its temperature, a constant temperature such as 4°C is adopted, for it is found that water at this temperature has its greatest density.

But the variation of density of water due to a change of temperature is very small, so that for all ordinary purposes where great accuracy is not required, it may be assumed that the weight of 1 c.c. of water is 1 gm. and that of 1 cu. ft. of water is 62·5 lbs. at any temperature.

Since gases are very light compared to water, the values of specific gravity of gases would be small fractions, if water is taken as the standard substance.

To avoid this, it is usual to adopt hydrogen at a standard temperature and pressure as the standard substance. The density of hydrogen at 0°C and 760 mm. of pressure is 0·0000896 gm. per c.c.

It is to be noted that the specific gravity of a body being a ratio of two weights, is a *pure number*, and does not depend on the units in which the weights are expressed, so long, as the same unit is used for the two. It merely expresses how much a body weighs as compared with water. Thus when it is said that the specific gravity of gold is 19, it is meant that, volume for volume, gold is 19 times as heavy as water.

The specific gravity may also be defined in other forms. Since the weight of a body is proportional to its mass, we may write

$$S = \frac{W}{W'} = \frac{Mg}{M'g} = \frac{M}{M'} \qquad \dots (88)$$

Where M is the mass of the body and M′ the mass of an equal volume of water at 4°C.

Again, if V is the volume of the substance, we have

$$S = \frac{M/V}{M'/V} = \frac{\rho}{\rho'} \qquad \dots (89)$$

Where ρ and ρ' are the densities of the substance and of water at 4°C respectively.

Since specific gravity is the ratio of two quantities of identical dimensions, it is a pure number having no dimensions. But density, being defined as the mass per unit volume, has the dimensions

$[M][L]^{-3}$. In the C.G.S. syestem, the average density of water is 1 gm. per c.c. Hence it is evident from eqn.(89) that the specific gravity of a substance is numerically equal to its density expressed in the C. G. S. system of units.

149. Measurement of Sp. Gr.—The specific gravity of a body is generally determined by the following methods which require the use of

(a) THE HYDROSTATIC BALANCE.

(b) HYDROMETERS.

(c) THE SPECIFIC GRAVITY BOTTLE.

(d) THE BALANCING LIQUID COLUMNS.

To measure the specific gravity of a substance we are mainly concerned with the determination of two weights,—the weight of the body and the weight of an equal volume of water. Now the weight of a body as determined by a balance in the air is not its true weight, for the body weighed as well as the counterpoise used in weighing will, according to Archimedes' Principle, be buoyed up in the air and will each suffer an apparent loss of weight equal to the weight of the air displaced. The loss in weight of the body will not evidently be, in general, equal to that of the counterpoise. But the effect arising from the difference is very small and is hence negligible. So in the expression for the specific gravity of a body we shall take the weight of the body in air in place of its true weight in vacuum.

It is to be noted also that in the following articles the expressions arrived at for the specific gravity of a body give the sp. gr. of the substance, relative to water at the temperature at the time of the experiment. Had the temperature of the water been 4°C, the true sp. gr. of the substance at 4°C would be given by S, as determined by the experiment. If, however,

the water is at any temperature t°C and if the density of the water at t°C is x times that of water at 4°C, it follows that the substance which is S times as dense as water at t°C, is Sx times as dense as water at 4°C.

The water which is used for the purpose of the determination of specific gravity should be pure distilled water free from air.

150. Determination of the Sp. Gr. of a Solid

(1) DIRECT METHOD

Expt. 34. Weigh the solid by a balance and let its weight be W gms.

Collect a quantity of water equal to the volume of the solid by the method of displacement of water (see art 20). Find the weight of the water thus displaced. Let it be W'.

Then the sp. gr. required $= W/W'$

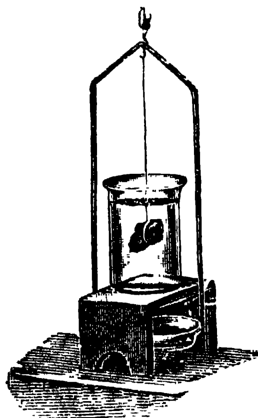


FIG. 145
Hydrostatic balance

(2) WITH THE HYDROSTATIC BALANCE.

The description of the hydrostatic balance has already been given in art. **145.**

(*a*) *The solid sinks in water and is insoluble in it.*

Expt. 35. Let W be the weight of the body in air taken in the usual way. Suspend the body by a fine wire from the hook of one side of the beam: and let it be *totally* immersed in a vessel of water. Let W′ be the apparent weight of the body in water (Fig. 145).

Then W — W′
= the upward thrust of the water on the body.
= wt. of the water displaced. (*by Archimedes' Principle*)
= wt. of an equal volume of water.

$$\therefore \text{ Sp. gr.} = \frac{W}{W-W'} \quad \ldots \quad \ldots \quad \ldots \quad (90)$$

This gives, as is remarked before, the specific gravity of the solid relative to water at the room temperature and is subject to correction for this.

(*b*) *Body insoluble in water but is less dense than it.*

In such a case, the body must be attached to another body, called a SINKER, of such a kind that the two together would sink in water. This complicates the method slightly.

Expt. 36. Find the weight of solid in air. Let it be W.

Weigh the sinker alone in water. Let this weight be W_1.

Next weigh in water the combination of the solid and the sinker fastened together. Let this weight be W_2.

Now suppose W_3 is the weight of the sinker in air. This weight need not be found in practice. Then

Wt. of water displaced by sinker $= W_3 - W_1$

Wt. of water displaced by combination $= W + W_3 - W_2$

$\therefore$ Wt. of water displaced by the solid alone

$$= (W + W_3 - W_2) - (W_3 - W_1)$$
$$= W - W_2 + W_1$$

$$\therefore \text{ Sp. Gr.} = \frac{\text{wt. of body}}{\text{wt. of equal vol. of water}}$$

$$= \frac{W}{W - W_2 + W_1} \qquad \dots (91)$$

We may obtain the expression for the weight of the water displaced by the solid from another point of view :—When the combination is immersed in water, the upward thrust of water displaced by the solid alone does not only balance W, the weight of the solid, but is greater than that, the excess causing a diminution in W_1, the weight of the sinker in water, of magnitude $W_1 - W_2$. In the absence of the sinker the body would have been floated up.

Hence the buoyancy on the *solid* in water

$$= W + (W_1 - W_2)$$

$$= W - W_2 + W_1$$

and the sp. gr. $= \dfrac{W}{W - W_2 + W_1}$

(c) *Solid soluble in water.*

If the solid be soluble in water, some other liquid must be chosen, in which the solid is insoluble. Let S be the specific gravity of the solid relative to this liquid, determined in the same general way as mentioned above. If S′ denotes the density of the liquid relative to the water at 4 C, then SS′ is the required specific gravity of the solid.

(3) WITH NICHOLSON'S HYDROMETER.

Hydrometer. A hydrometer is an instrument which is designed to float vertically in any liquid and constructed to determine mainly the sp. gr. of liquids. There are various forms of the hydrometer ; but they can all be put in either of two classes. In the type of the hydrometer known as the *Variable Immersion Hydrometer*, the specific gravity of a liquid is determined by the depth to which the hydrometer sinks when floating in the liquid ; in the type known as the *Constant Immersion Hydrometer*, the hydrometer is always immersed to the same depth in the liquid but carries different weights.

Nicholson's Hydrometer—This is a constant-immersion hydrometer and is the only one in common use. It is also used to find the sp. gr. of a solid. It consists of a hollow body to which is attached a thin stem C, supporting a small pan B above, on which weights can be placed. Below the body hangs a second pan D. This is loaded with mercury

or lead, so that the instrument may float vertically in a liquid. On the stem there is a mark to which the instrument is made to sink in the liquid in which it is floating. Thus whatever be the liquid the volume displaced is always the same.

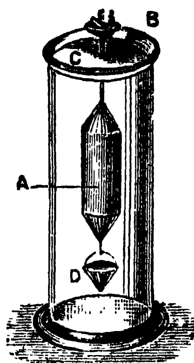


Fig. 146
Nicholson's Hydrometer

To determine the sp. gr. of a solid substance by Nicholson's Hydrometer, proceed as follows :

Expt. 37. Float the hydrometer in water in a tall and wide jar (Fig. 146). Place weights on the upper pan to sink the hydrometer to the index mark on the stem C Let W_1 be this weight. Replace the weights in the weight-box and place on the pan the piece of solid of which the specific gravity is to be determined. The solid must not be so heavy as to sink the instrument to the index mark or lower. Now place weights on the upper pan to sink the instrument to the index mark. Let this weight be W_2.

∴ The weight of the solid $= W_1 - W_2$.

The solid is next placed on the lower pan within the water where it displaces its own volume of water. As the solid is now acted on by the buoyancy of the liquid displaced, the hydrometer rises a little. Place additional weights on the upper pan until it again sinks to the mark. Let W_3 be the total weight on the pan.

∴ The weight of water displaced by the solid when placed in the lower pan $= W_3 - W_2$.

$$\text{Hence sp.gr. of the solid} = \frac{W_1 - W_2}{W_3 - W_2} \qquad \ldots (92)$$

It is to be noted that this method is not so accurate as the other methods are, which involve the operation of weighing by means of a balance.

(4) WITH THE SP. GR. BOTTLE.

Specific Gravity Bottle.—This is a bottle capable of holding a known volume of a liquid. Two forms of bottle are shown in fig. 147. In (i) it is a small flask fitted with a ground glass stopper made from a short length of a thick-walled tubing of a very fine bore. The bottle is filled to the top of the neck with any liquid and the stopper is then pushed home so as to cause the surplus liquid to escape by the hole in the stopper leaving the bottle completely filled. In (ii) the bottle has a narrow neck which is open and has a fine mark on it. When in use, the bottle is always filled exactly up to this mark.

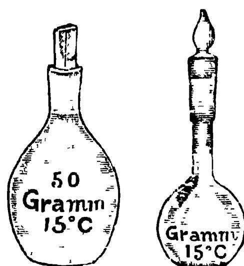


Fig. 147
Specific Gravity Bottles

The specific gravity bottle is used to find the specific gravity of a liquid ; it is also used for finding the specific gravity of a solid in the form of powder or in small fragments that can be inserted into the bottle, *e.g.*, shorts, sand, insoluble powders, metal filings, etc.

Expt. 38. Weigh the solid : let the weight be W. Fill the bottle with water and place it with the solid on the same pan of a balance : let the combined weight be W_1. Now place the solid *inside* the bottle and fill it up with water, taking care that no air-bubble sticks to the sides of the solid : weigh it. Let this weight be W_2. Evidently, W_2 is less than W_1, as the solid expels water of its own volume.

$\therefore$ Weight of water displaced $= W_1 - W_2$

$$\therefore \text{ sp. gr. of solid } = \frac{W}{W_1 - W_2} \quad \ldots \quad (93)$$

This result must, if necessary, be corrected for the temperature of water as explained above.

151. Determination of the Sp. Gr. of a Liquid.

(1) WITH THE SP. GR. BOTTLE.

Expt. 39. The bottle is made throughly clean and dried by blowing hot air into it. Weigh it carefully when empty: let this weight be W_1. Fill it with water and again weigh it. let W_2 be this weight. Finally fill it with the given liquid and weigh it: let W_3 be this weight.

Then Wt. of liquid filling the bottle $= W_3 - W_1$

and Wt. of water do do $= W_2 - W_1$

Hence the sp. gr. of the liquid relative to water at the room temperature is given by

$$S = \frac{W_3 - W_1}{W_2 - W_1} \quad \ldots \quad (94)$$

(2) WITH THE HYDROSTATIC BALANCE

A body which sinks in and is insoluble in both the given liquid and water, is to be taken. According to Archimedes' Principle the apparent loss of weight of a sinker in any liquid gives exactly the weight of the displaced liquid. Hence, if the *same* sinker be weighed in the given liquid and in water the apparent loss of its weight in the liquid gives the weight of the liquid of the same volume as of the sinker, and that in water gives the weight of an *exactly equal volume* of water.

Let the weight of sinker in air $= W$
Do do in water $= W_1$
Do do in the liquid $= W_2$
Then the Wt. of a quantity of *liquid* equal in volume to the sinker $= W - W_2$
and the weight of a quantity of *water* equal in volume to the sinker $= W - W_1$

$$\text{Hence sp. gr.} = \frac{W - W_2}{W - W_1} \quad (95)$$

(3) WITH THE HYDROMETERS.

The Constant Immersion Hydrometer :—A description of the hydrometer has already been given in the

previous article. NICHOLSON'S HYDROMETER which belongs to the class of the *Constant Immersion Hydrometers* may be used to compare the specific gravities of two liquids. When the hydrometer floats in any liquid immersed exactly up to the index-mark on the stem, the weight of the instrument together with the weights on the pan is, by the principle of Archimedes, equal to the upward pressure, *i.e.*, to the weight of the displaced liquid.

Let W be the weight of the hydrometer.
W_1 be the weight necessary to sink it up to the index mark in the liquid.
W_2 be the weight required to sink it up to the mark in water.

Then the weight of liquid displaced $= W + W_1$
and that of water displaced $= W + W_2$

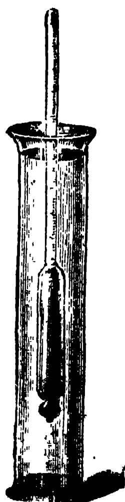


Fig. 148
Hydrometer

Hence

$$\text{sp. gr. of the liquid} = \frac{W + W_1}{W + W_2} \qquad (96)$$

The Variable Immersion Hydrometer:– A common form of a hydrometer of the variable immersion type is shown in fig. 148. It consists of a hollow glass stem ending below in a glass bulb weighted with mercury, so adjusted as to make the instrument float in a liquid with its stem vertical. The stem is a cylindrical tube and carries a paper-scale inside, the graduations of which are meant to give directly the specific gravity of liquids in which the instrument is immersed. This type of hydrometers is much in use for commercial purposes.

It is evident that the instrument, when allowed to float in a liquid, will sink the deeper, the less the specific gravity of the liquid, since the weight of the

liquid displaced must always be equal to the weight of the instrument. The calculations by which the markings on the scale are obtained are based on this fact ; in practice, however, the instrument-makers calibrate the scale by immersing the instrument in liquids of known specific gravities.

If the hydrometer sinks to the mark, say 1 in the stem, in water at 4 C, the marks for specific gravities of the liquids denser than water will be below the mark 1, and those for the lighter liquids above the mark 1. Hence a hydrometer to show the specific gravities of liquids of all densities would have to be inconveniently long. Hydrometers are, therefore, usually made in *sets* to be used separately, for liquids lighter and heavier than water. In the former, the mark 1 (generally marked 1000 to avoid decimals) is pretty near the bottom of the stem and the graduation running up the stem decreases towards the top : in the latter, the mark 1, is near to the top of the stem and the graduation increases down the stem. The scale is usually adjusted to indicate a change of 0·001 in the specific gravity of a liquid, and each tenth division is marked. Thus, if a hydrometer floats in a liquid immersed to a point marked 850 in the scale on the stem, the specific gravity of the liquid is 850/1000 or 0·85. The divisions on the scale are not equal, but decrease in length as the bottom of the stem is approached.

Twaddell's Hydrometer, used in England and *Beaume's Hydrometer*, used in the Continent are both of the variable immersion type and are graduated in arbitrary scales. Hydrometers are often specially graduated for specific purposes, thus an *alcoholometer* for determining the strength of alcoholic liquors, a *urinometer* for urine, *a lactometer* for milk, etc.

(4) BY THE LIQUID COLUMN METHOD.

The specific gravity of a liquid may be determined

by balancing a column of the liquid hydrostatically against a column of water, and then comparing the heights of the two balancing columns.

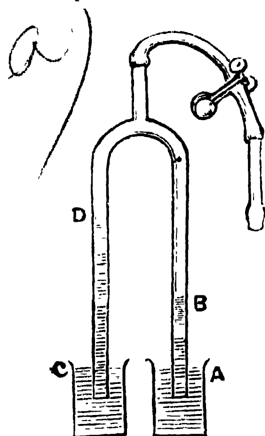


FIG. 149
Hare's Apparatus

The U-tube (in Fig. 132) may be used when two liquids of which the densities are to be compared, do not mix, *e.g.*, mercury and water. It has already been shown in art. 138 that when two liquids are in equilibrium, their heights above their common surface vary inversely as their densities.

$$\text{or} \quad \frac{h}{h'} = \frac{\rho'}{\rho}$$

In the case of two miscible liquids such as water and alcohol, the same U-tube may be employed, but a third liquid such as mercury which does not mix with either and at the same time heavier than both, is to be used. Mercury is first put in the U-tube; water is then poured into one limb and alcohol into the other, till the level of mercury is the same in both the limbs. The principle mentioned above is then to be applied.

But a more convenient apparatus for comparing the densities of two liquids which will mix, is what is usually known as the HARE'S HYDROMETER (Fig. 149). It is simply an inverted U-tube dipping into two beakers each containing a liquid. At the top there is an opening to which is attached a short length of an India-rubber tubing provided with a clip and carrying a glass tube at the other end which is used to draw air out of the tubes. As the air is

partially sucked out from the tubes, the pressure of the air in the tubes decreases. Urged by the external atmospheric pressure on the liquid surfaces in the beakers outside the tubes, the liquids are forced up into the tubes. The ratio of the heights of the liquid columns in the two limbs above the free surfaces of the liquids in the respective beakers gives the ratio of the densities of the two liquids.

Let ρ and ρ' be the densities of the two liquids in CD and AB respectively; h and h' the heights of the liquid columns, each measured from its level within the beaker, and P the pressure of air at the top of the U-tube.

then Pressure at C = pressure at A = atmospheric pressure.

$$\therefore \quad g\rho h + P = g\rho' h' + P$$

$$\text{Hence} \quad \rho h = \rho' h'$$

$$\text{Or} \quad \frac{\rho'}{\rho} = \frac{h}{h'} \qquad \dots (97)$$

$$\therefore \quad \rho' = \rho\frac{h}{h'}$$

The method of comparing specific gravities by the balancing liquid column method is not ordinarily an accurate method.

Exercise XVI

N. B.—In working out Examples on specific gravity the established formulæ should never be used. The successive steps in the argument must be worked out from the first principles.

1. A Nicholson's hydrometer weighs 200 gms. and requires 50 gms. in the upper pan to sink it to the fixed mark in water; what weight must be added to or subtracted from the weights in the upper pan to bring it to the fixed mark when it is placed in a liquid of specific gravity 1·2? [C. U.—1911]

2. A piece of metal weights 100 gms. in air and 88 gms. in water. What would it weigh in a liquid of sp. gr. 1·5? [C. U.—1915]

3. Describe a method of determining the sp. gr. of a liquid.

A Nicholson's hydrometer sinks to a certain mark in a liquid of sp. gr. 0·6; but it takes 120 gms. to sink it to the same mark in water. What is the weight of the hydrometer? [C. U.—1918]

4. A piece of glass weighs 8·6 gms. in air, 5·85 gms. in water, and 6·4 gms. in Alcohol. Find the sp. gr. of Alcohol. [C. U.—1920]

5. Define specific gravity of a body.

Describe in detail how the specific gravity of a block of alum can be actually determined. [C. U.—1921]

6. The specific gravity of ice is 0·918 and that of sea-water is 1·03; what is the total volume of an iceberg which floats with 700 cubic yards exposed? [C. U.—1923]

7. The apparent weight of a piece of platinum in water is 60 grammes, and the absolute weight of another piece of platinum twice as big as the former is 126 grammes. Determine the specific gravity of platinum. [C. U.—1924]

8. How do you find the specific gravity of a solid lighter than water.

A piece of cork whose weight is 19 grammes is attached to a bar of silver weighing 68 grammes and the two together just float in water. The specific gravity of silver is 10·5. Find the specific gravity of cork. [C. U.—1925]

9. The metal sodium is lighter than water. How would you measure its specific gravity?

A metal tube 104 cm. long, 4·1 cm. in external diameter, 3·5 cm. in internal diameter, weighs 100 grams: of what metal would you judge it to consist and why? [Pat. U.—1921]

10. Find the specific gravity of a solid substance from the following data:—A flask, which when filled with water, weighs altogether 410 grammes, has 80 grammes of the solid introduced and being then filled with water weighs 470 grammes. [C. U.—1926]

11. A mixture is made of 7 c.c. of a liquid of specific gravity 1·85 and 5 c.c. of water. The specific gravity of the mixture is found to be 1·615. Determine the amount of contraction. [C. U.—1927]

12. Explain how you would determine the specific gravity of a solid by a specific gravity bottle.

60·3 gm. have to be placed on the pan of a hydrometer to sink it to the mark in water and 6·8 gm. only in Alcohol. If the hydrometer weighs 200 gms. what is the specific gravity of Alcohol ? [C. U.—1931]

13. Describe a Nicholson's hydrometer, and explain how you would determine the specific gravity of a liquid with its help.

1 cc. of lead (sp. gr. 11·4) and 21 c.c. of wood (sp. gr. 0·5) are fixed together. Show whether the combination will float or sink in water. [C. U.—1933]

14. A specific gravity bottle weighs 14·72 gms. when empty, 29·74 gms. when filled with water, and 44·85 gms. when filled with a solution of common salt. What is the specific gravity of the solution ? [C. U.—1934]

CHAPTER XIX

MOLECULAR MOTIONS AND FORCES IN LIQUIDS.

152. Molecular Motions in Liquids—Experiments with liquids show that their constituent molecules must be moving continuously from place to place. Evidence of this is found in the very familiar facts of *evaporation* and also in the facts of *diffusion* and *expansion of liquids*.

Further, to account for the phenomena connected with capillarity and surface tension (arts. 154 and 155), it is necessary to assume that the molecules of a liquid are so close together that the effect of their mutual attraction must be taken into account. In solids, we have seen that this mutual attraction gives rise to *cohesion*: in gases, the average distance between the neighbouring molecules is supposed to be so great, that the effects of this mutual attraction may be left out of account.

Evaporation. When a saucer full of water is placed in an open space, the water is observed to diminish gradually by evaporation. The molecules of water pass continuously into the open space above until the dish is left dry. This phenomenon is difficult to be explained, unless it be assumed that the molecules in the liquid are in motion. During their motion the liquid molceules come into frequent collisions with one other, due to which it is reasonable to consider that in a liquid at a constant temperature a molecule may be moving with a greater velocity at one moment than at another. At any instant some

molecules are, therefore, moving more rapidly than the rest. Such molecules may, on account of their great velocity, break away from the attraction of their neighbours and escape into the space above.

If it be correct to suppose that the *heat* contained in a body is a measure of the kinetic energy of its constituent molecules, it must follow that an increase in temperature means an increase of the velocities of the molecules. The components of the velocities perpendicular to the liquid surface thus increase correspondingly so that the chance of escape of a molecule which may happen to be moving at any instant towards the liquid surface also increases ; in other words, evaporation ought to take place more rapidly at higher temperatures than at lower ones. This is, of course, known to be true from our daily observations.

If evaporation takes place in *a closed space*, some of the molecules of the vapour, after wandering about for a time, will strike against the surface of the liquid, and again pass into it. Other molecules will, however, be escaping, and it is clear that after a certain time a state of equilibrium will be reached, in which as many molecules return to the liquid in a second as leave it in that time. The vapour is then said to be in a *saturated* condition.

Expansion.—The same supposition explains the fact of expansion of liquids too. If heat be applied to a liquid contained in a bulb provided with a stem, the level of the liquid in the stem is observed to rise. This is a further evidence of the fact that the velocity of motion of the molecules of a liquid increases with an increase of the temperature of the liquid.

Diffusion.—Again, in the phenomenon known as *diffusion of liquids*, one liquid spreads into another by molecular motion without the aid of external forces. If a concentrated solution of copper sulphate

be placed in a beaker and water be very gently poured on it down the sides of the beaker, the two liquids will form two separate layers, the horizontal surface of separation between them being quite sharp and distinct. After sometime, however, it will be found that though the liquids are undisturbed, the blue colour extends upwards indicating that some molecules of the dissolved salt pass upwards ; on the other hand, the deep blue colour of the concentrated solution becomes fainter and fainter as it is diluted by the molecules of water passing downwards into the solution.

Osmosis.—Further, the supposition does well explain the *Osmotic* phenomenon in which two liquids which will mix, diffuse into each other even when they are separated by a membrane or a porous diaphragm.

Expt. 40. Fill a sheep's bladder with a strong brine solution. Tie it tightly and then leave it in pure water. It will be found after sometime that the bladder is gradually extended to the bursting point ; also the liquid outside the bladder has a salty taste.

It thus appears that the molecules of both the liquids pass through the diaphragm in opposite directions but with unequal velocities, the lighter liquid, *viz.*, the water passing in more rapidly than the salt passing out. Accordingly there is an accumulation of water and hence an increase of pressure inside the bladder.

153. Molecular Forces in Liquids.—It has already been mentioned that the mutual attraction of molecules within bodies is powerful only when the molecules are not separated by sensible distances. In liquids, the molecular forces are very small compared to those in solids, but they are not negligible.

Expt. 41. Support a smooth glass plate horizontally from one arm of a common balance and counterpoise it. Then

let the lower surface of the plate come in contact with the clean surface of water contained in a vessel. Now try to detach the plate from the water surface. The force required for this purpose will be found to be rather considerable; the weights necessary to put on the other pan to detach the plate will show this.

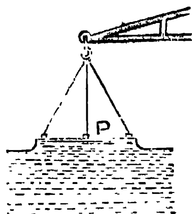


Fig. 150

Measurement of cohesion of water

Since a thin layer of water is found touching the glass plate, it is evident that the force applied to pull the plate up has been spent in separating a thin layer of water away from the rest of the water, and hence it is a measure of the cohesion in water. Secondly, the force of adhesion between the glass and the water is greater than the force of cohesion in water.

In the case of mercury, the glass will not be found to be wet, showing that the cohesion in mercury is greater than the adhesion between glass and mercury.

154. Surface Tension.—Since every molecule of a liquid is pulling every other molecule, a molecule such as A (Fig. 151), situated well within the mass of a liquid, will be attracted by the neighbouring molecules equally in all directions; whereas any molecule such as B, situated near to the surface or on it, is attracted by a resultant force directed towards the inside of the liquid mass and perpendicular to the surface. Due to this, the surface molecules of the liquid have a tendency to move towards the interior of the mass so that the latter may have the smallest possible surface compatible with its volume. Owing to this tendency of the surface to contract, any given quantity of a liquid behaves as if it were surrounded by a thin elastic membrane which

is stretched by a definite force characteristic of the liquid. This force is called the surface tension of the liquid.

It follows that any mass of liquid would assume a spherical form, as the sphere is the geometrical figure which has the smallest area for a given volume, provided that we relieve the mass from the action of gravity and other outside forces which, in ordinary cases, mask the presence of the cohesive forces.

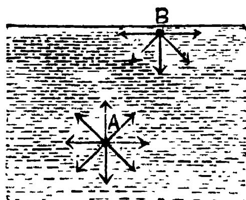


Fig. 151
Forces acting on a molecule
A—inside the liquid
B—near the surface

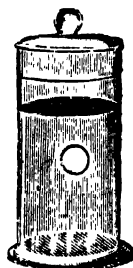


Fig. 152
Globule of oil freed
from the action
of gravity.

Expt. 42. Prepare a mixture of alcohol and water such that it has the same density as that of olive oil. Insert a large drop of the oil beneath the liquid surface by means of a pipette. The oil will be seen to float as a perfect sphere within the liquid mass (Fig. 150).

When the oil floats inside a liquid of the same density as its own the force of gravity has no influence on it, and the drop owing to the cohesion draws itself up into a spherical form.

Again, in small masses of liquids, the force of gravity is negligibly small compared to the cohesive forces; and in such cases the special form of the mass is frequently apparent. Thus very small globules of mercury splattering on a table, rain-drops, dew-drops, etc., have more or less spherical forms.

That the thin surface-layer of a liquid acts as a stretched membrane under a uniform tension in all directions, may also be understood from the following experiment.

Expt. 43. Let a sewing-needle be slightly greased and be placed very carefully on the surface of water in a dish. Though it is nearly eight times as dense as water, it will be found to float. If the needle be previously magnetized, it can be made to move about by means of a magnet held near to it.

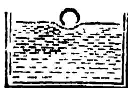


Fig. 153
A needle
floating on water



Fig. 154
An insect walking
on water

Observe the surface in the neighbourhood of the needle ; it shows a slight depression as seen in fig. 151.

The floating of the needle in Expt. 43 seems to be a contradiction of the condition of flotation as deduced from the law of Archimedes (art. 144). But the explanation is quite clear, the depressed portion of the liquid surface has a tendency to straighten out into a flat surface ; the weight of the needle is supported by the vertical components of the surface tension round the edge of the depression. Had the water wetted the needle, as it would have done in the absence of any grease round it, water would have risen about the needle ; the tendency of the liquid surface to flatten out would then have pulled it down.

The abve experiment explains the phenomena of some insects walking on the surface of water without sinking (Fig. 152).

The fact that the surface of a liquid behaves as if it is subjected to a tension is further illustrated by the behaviour of liquid films.

Expt. 44. Take a bent wire ABC (fig. 155). Allow a thin straight wire simply to rest against this. Get a soap film

enclosed in the portion DBE. It will be found necessary to exert a small force (P) to prevent DE from being drawn up due to the contractility of the soap film.

Expt. 45. Dip a flat wire ring in a soap solution and withdraw it. A thin film of the solution will be found stretched across it. Moisten a small loop of thread with the solution and place it gently on the film. It is seen to retain any irregular form that may be given to it (fig. 156). Now break the film within the loop; the loop immediately takes up the circular form (fiig. 157).

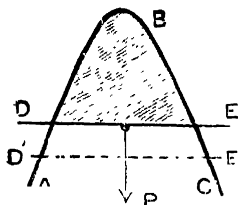


Fig. 155

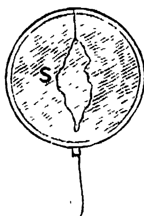


Fig. 156

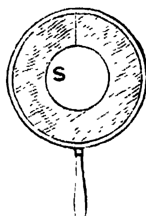


Fig. 157

To illustrate the contractility of a soap film

The tendency to contract of the film outside the loop, is now no longer balanced, as the outer film has vanished. Since the film outside tends to assume the smallest possible surface, the area inside the loop becomes as large as possible, and the circle is the figure which has the largest possible area for a given perimeter.

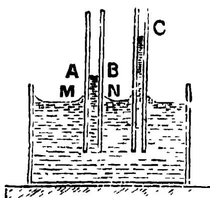
155. Capillarity.—It was stated in art. 136 that in general, a liquid stands at the same level in any number of communicating vessels. This rule is, however, subject to exceptions in the case of tubes of small diameter (or *capillary* tubes, as they are called from *capillus*, a hair) and in the neighbourhood of the sides of the vessel in which the liquids are contained.

Expt. 46. Dip glass tubes of different bores in water. As water wets glass, the surface of the water round the line of its contact with glass, inside the tube, outside the tube and

round the inner surface of the containing vessel is not horizontal but is concave upward (fig. 158). It is to be noted also that the water rises higher in the tubes than in the vessel. and the finer the bore, the greater is the height to which it rises.

Now replace the water by mercury. The effects are found to be just opposite. Mercury does not wet glass and its surface round the line of contact with glass is found to be convex upward. Further, mercury is depressed in all the tubes, the depression being greater in proportion as the bore of the tube is smaller (fig. 159). To see this, bring the tube close to the side of the glass vessel containing mercury. The depression is more easily observed with a U-tube.

Measurement of the diameter of the bore of each tube and of the capillary elevation or depression in it shows that the latter is inversely proportional to the former.



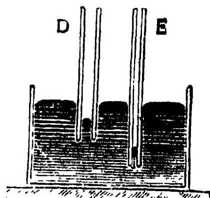


Fig. 158 Fig. 159

Capillary elevation and depression of liquids

The phenomena described in the above experiments are examples of what is known as capillarity. Such experiments have established the following laws of capillarity :

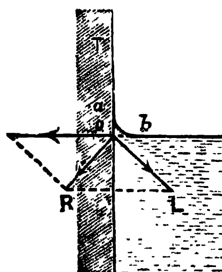
1. When a capillary tube is placed in a liquid, the liquid is raised or depressed according as it does or does not moisten the tube.

2. The elevation in one case, and the depression in the other, are inversely poportional to the diameters of the tube.

3. *The height varies with the nature of the liquid and decreases as the temperature rises.*

We now proceed to give an explanation of the effects observed above :—

Suppose the horizontal surface of water meets the glass side at O (Fig. 160). Let us consider a small portion of the liquid surface at O. As water wets glass, the force of adhesion between glass and water will pull the liquid particles at O in the direction OG : again, the resultant of the cohesive forces within the liquid will pull the same particles in the direction OL. As the former force greatly exceeds the latter, the resultant of OG and OL will be some force OR which lies to the left of the vertical OT. Now since a liquid surface always sets itself at right angles to the resultant force acting on it (art. 122), the water surface at O must rise up against the wall, and present a concave surface upward. Further, we have already seen that in small masses of liquids, the force of cohesion preponderates over the force of gravity on it.



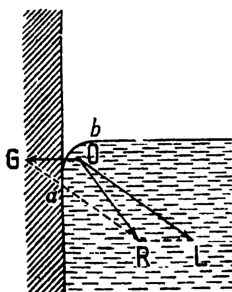


Fig. 160 Fig. 161
Ascension and depression of a liquid surface near a wall

Conversely, if the cohesive force OL is stronger

than the adhesive force OG, as is the case when mercury comes in contact with glass, the resultant OR (Fig. 161), will fall to the right of the vertical through O, in which case the liquid must be depressed at O and present a convex surface about O.

The above facts combined with the fact that the exposed surface of a liquid always tends to reduce its area explains the ascension and depression of liquids in capillary tubes. In fig. 158 the curved surface of water inside the tube tends to become flat due to surface tension. But as soon as it begins to be flat, the forces of adhesion again elevate it at the edges It is thus evident that the water must continue to rise within the tube, until the tendency of the surface to move up is balanced by the weight of the column of water thus raised. We have then the upward pull due to the surface tension along the line of contact = weight of the raised liquid column acting downward.

If r is the radius of the bore of the tube and the surface tension is T dynes per cm. along the line of contact, the force due to surface tension which acts vertically upwards is $2\pi r T$.

And the weight of the liquid column of elevation h above the free surface of the liquid outside the tube is $\pi r^2 h \rho g$

$$\therefore \quad 2\pi r T = \pi r^2 h \rho g$$

$$\text{i. e.,} \qquad T = \frac{rh\rho g}{2}$$

In the case of water, $\rho = 1$

$$\therefore \qquad T = \frac{rhg}{2} \qquad \dots (98)$$

Now if the diameter of one tube be half of that of another, the total upward force due to the surface tension is reduced to one-half, since the circumference ($2\pi r$) of the liquid surface is reduced by one-half. But

the weight of the liquid column of the same height as that in the wider tube, is only one-fourth of the latter since the volume of the liquid varies as the square of the radius. Hence, for the equilibrium of the liquid column in the narrower tube, the height of the liquid column must be twice that obtained in the wider tube.

In the case of mercury, it must fall owing to the tendency of the convex surface of the liquid in the capillary tube to straighten out. The fall continues until this tendency is balanced by the hydrostatic pressure at O.

Instances of capillartty are very common and play rather an important part in our everyday life. The rise of oil in wicks of oil-lamps, of melted tallow in the wicks of a candle, the flow of blood through the capillary tubes within the body, the rise of sap in plants, the retention of water in a piece of sponge, the rise of ink in the narrow slit of a pen, the soaking up of ink by the blotting paper, the rapid absorption of water by a dry brick partially immersed in it are excellent illustrations of capillarity.

Exercise XVII

1 What do you mean by "surface tension"? Describe a phenomenon which exhibits the surface tension of a liquid.

2. Explain the term "capillarity."

3. If a capillary tube is immersed in a liquid, the liquid inside the tube rises above or is depressed below the outer surface of the liquid according as it does or does not moisten the tube. Explain why?

CHAPTER XX

PROPERTIES OF GASES : THE ATMOSPHERE

156. Gases.—It has already been stated that gaseous substances possess a number of properties in common with liquids and both are therefore commonly classed as fluids. Gases like liquids, transmit pressures equally and in all directions according to Pascal's Law ; like them, they possess elasticity of volume only and not elasticity of shape. They differ essentially from the liquids in that they are very much lighter and are very compressible and capable of indefinite expansion (art. 121).

The volume of a given solid or of a given mass of liquid is a characteristic property of the solid or the liquid considered. But a given mass of gas can have any volume whatsoever, this latter only depending on the vessel which contains it, for the chief charcteristic of a gas is that it completely fills the space to which it has access.

The change of volume of a solid or a given mass of liquid for a given change of pressure or temperature is quite negligible in comparison with the change in volume that a given quantity of gas undergoes under the same change of conditions.

157. Weight of Gases.—Gases are very much lighter than solids or liquids. In fact, the mass of a given volume of any gas is so small in comparison with an equal volume of any known solid or

liquid that its weight is not sensible to us under ordinary circumstances. The fact that air, which is the type of gas most commonly known to us, has weight was first proved by OTTO VON GUERICKE, the inventor of the air-pump in 1650, and may be shown as follows :

Expt. 47. A thin glass globe, four or five inches in diameter is provided with a stop-cock and a nozzle by which it can be connected to an air-pump (fig. 162). Exhaust the globe as far as possible and determine its weight by means of a delicate balance. Now admit air by opening the stop-cock and again weigh the globe. The weight in the second case will be found to be greater and the increase in weight is due to the air admitted.

The above fact may also be proved by the following experiment in which the use of an air-pump is not required.

Expt. 48. Take a fairly large glass flask and close it tight with an India-rubber stopper, through which passes a short

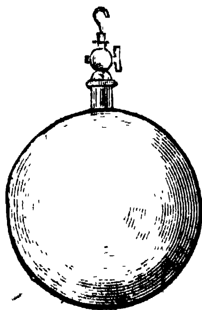


Fig. 162

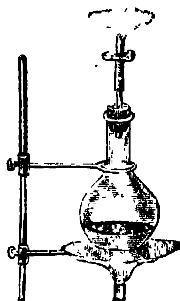


Fig. 163

To demonstrate that the air has weight

tube carrying a rubber tube and a pinch-cock (fig. 163). Put a little water in the flask, open the pinch-cock and boil the water. As steam comes out, it sweeps out the greater part of the air in the flask. After some time close the pinch-cock

quickly and remove the flask to a side to let it cool. Determine the weight of the flask when it is cool.

Now open the clip: the air will be heard to rush in with a hissing sound. Re-weigh the flask carefully. The increase of weight in the second case is due to the air that has entered the flask.

Measure the water in the flask by means of a graduated cylinder: fill the flask with water up to the position occupied by the bottom of the stopper, and measure its volume. The difference of these volumes gives approximately the volume of the air which entered the flask.

From the latter part of Expt. 48 the weight of a litre of air at the temperature and pressure at the time of the experiment may be roughly found. Under standard conditions *viz.*, when the temperature is 0 C and the pressure is due to a head of 76 cms. of mercury, a litre of dry air weighs 1·293 gms. Thus under these circumstances the density of air is ·001293 gm. per c c.

158. Pressure of the Atmosphere.—It has already been stated that the atmosphere encircles the earth as a spherical layer of air which extends, as an appreciable atmosphere, up to very great heights above the surface of the earth. The surface of the earth and bodies thereon are subjected to the pressure produced by the weight of the overlying air. This pressure in the air is called the **atmospheric pressure**. It is subject to the fundamental laws of fluid pressure given in chapter XV.

The pressure in the air at any level being due to the weight of the air above, this pressure must evidently be the greatest at the surface of the earth, and must decrease as we ascend higher in the atmosphere; it has the same value for all points in the same horizontal layer, provided that the air is in a state of equilibrium.

As a gas is easily compressible under pressure the density of the air too is the greatest at the surface

of the earth, and decreases as the height above the surface increases. Whenever there is an inequality of density at a given level due to local conditions, wind must ensue.

Living as we do at the bottom of a deep sea of the air, we are not sensible of the pressure existing in the air around us, because it acts with an equal value in all directions. In order to make the atmospheric pressure manifest its effects, it must be made to act upon bodies from one side only.

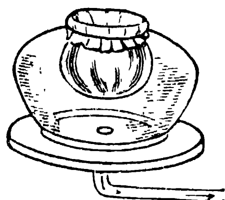


Fig. 164

Crushing force of the atmosphere

Expt. 49. Stretch an India-rubber membrane air-tight over one end of an open receiver. Grease the other end, and press it on the plate of an air-pump. As the rubber remains flat, the pressure is evidently the same on both sides of it. Exhaust the air from inside the vessel. The membrane is depressed more and more until it finally bursts under the pressure of the above (fig. 164); a loud report is caused by the sudden entrance of the air.

The experiment may be performed by placing the plain of one hand closely over the mouth of the open receiver. On working the pump, the weight of the air will be felt at once. As the exhaustion is carried far, it becomes difficult to lift up the hand. Further, the pressure of the air in the tissues of the palm is no more counterbalanced by that of the air inside the receiver; hence the palm of the hand swells giving a painful sensation.

Expt. 50. Put a small quantity of water into a can of thin tin sheet and boil the water briskly for some time so that the air in the can is expelled by the steam. Now close the can with a good well-fitting cork while the boiling is still going on. Cool the can in a sink by pouring water upon it. The can collapses.

As the can cools, the steam inside it condenses and a partial vacuum is produced inside the can. The walls of the can being not strong enough to withstand the great excess of external atmospheric pressure are crushed inwards (fig. 165).

Expt. 51. Fill a glass tumbler completely with water. Cover its mouth with a sheet of thick paper. Keeping the paper in position with one hand, invert the tumbler with the other. On withdrawing the hand which held the paper, the water will be found not to fall, both water and paper being kept in position by the superior pressure of the atmosphere acting in an upward direction (fig. 166). The object of the paper is to present a flat surface of water, as otherwise the mass of water would divide, allowing the air to enter.

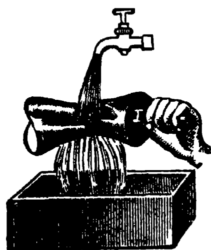


Fig. 165

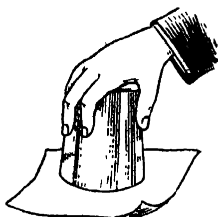


Fig. 166

Expt. 52. Take a bottle with a very narrow neck. Fill it with water and invert it. Not a drop will be split out.

The surface-tension of the drop of water at the mouth of the bottle plays the part of the sheet of paper in Expt. 51, in other words, it prevents the breaking up of the liquid.

Fig. 167 represents a tin-can with a perforated bottom. Its neck is fitted with a cork through which runs a hole. Fill the can with water. So long as the hole in the cork is closed with a finger, water will not come out. Water flows out only when communication with the outside air is allowed by opening the hole in the neck.

Fig. 168 represents a toy, called a *Magic Bottle*.

It is an opaque bottle of sheet iron or guttapercha, containing within it five small vials. Each vial, at its upper part, has a tube which passes up the neck of the bottle ; and at its lower end it has another tube terminating in a small hole on the side of the bottle. The five vials are filled with five different liquids. The operator closes the holes by the five fingers of his hand, and pours out at pleasure any liquid out of the bottle by cleverly uncovering the corresponding hole.

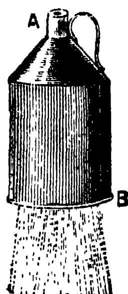


FIG. 167
A perforated tin-can

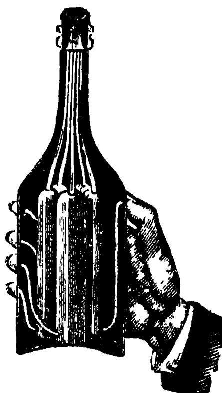


FIG. 168
A magic bottle

The effect of the pressure due to the atmosphere was demonstrated by Otto Von Guericke by means of two hollow metal hemispheres. As Guericke was burgomaster of Magdeburg in Prussia, the experiment has always been called the experiment of **Magdeburg Hemispheres**. The hemispheres fit so closely together as to be air-tight, and one of them is provided with a screw to be fitted on to an air-pump. Rings are attached to both the hemispheres at the extreme ends (Fig. 169).

Expt. 53. Grease the edges of the two hemispheres and put them together ; note that they are pulled apart easily so

long as they contain air. Put this together again ; exhaust the air from within them, and close the stop-cock. Note that a very great force is now needed to separate the two hemispheres.

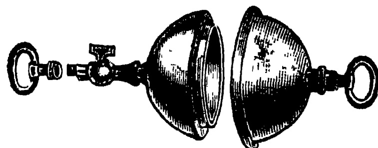


FIG. 169.—Magdeburg Hemispheres.

In one of Guericke's original experiments in 1654, it is said that hemispheres of 2 ft. diameter were used, and that a team of 12 horses, six on each side, was required to pull the hemispheres apart !

The Vacuum Automatic Brake.—We are all familiar with the vacuum automatic brake, or simply vacuum brake, fitted in railway trains. The action of the vacuum brake depends upon the tremendous pressure exerted by the atmosphere. Under each carriage there is a **brake cylinder** in which moves a closely fitting air-tight piston.

The piston rod is connected to a system of levers in such a way that the up and down motion of the piston is converted into a to and fro motion of the brake-blocks which press against the wheels. Each cylinder is connected to a **train-pipe** which runs beneath each carriage ; these pipes are joined together by flexible air-tight couplings. The front end of the train-pipe is fitted with a steam ejector by means of which the air can be almost completely exhausted from the whole of the train pipe. When this is done the piston in each cylinder is drawn down to its lowest position and the brakes are off. This is the normal state of affairs. When the driver stops the working of the ejector and air is admitted into the train-pipe, the pressure of the air forces up the pistons

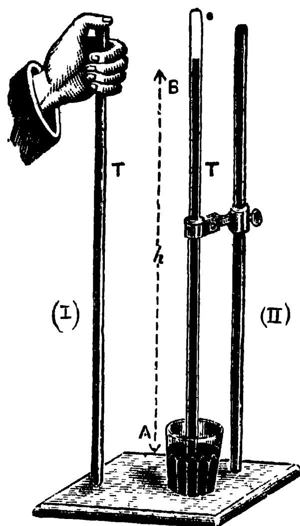
in the brake cylinders and the brake blocks are pressed hard against the wheels thus stopping their motion almost immediately. The guard can also admit air into the train pipes by a suitable mechanism so that the control of the brakes is in the hands of both the driver and the guard.

15. Rise of Liquids in Exhausted Tubes.—Before the time of Galileo the rise of water in a tube when the air is exhausted from its upper end, as in pumps and siphons, was explained by the supposition that *Nature abhors a vacuum*. In 1640, Galileo's attention was drawn to the fact that some pumps erected at the garden of the Duke of Tuscany near Florence designed to draw water from a depth of 56 ft. cannot work; the water rose about 30 ft. but would rise no higher demonstrating as it were, that Nature's abhorrence had its limits. Galileo seems to have [illegible]ected that the pressure of the air was responsible for the phenomena, but he died in 1642 without being able to prove it. The true explanation was, however, given by his friend and pupil, Torricelli (1608-1647), who took up the question and continued the investigation after Galileo's death.

16. Torricelli's Experiment.—Torricelli argued that if water rose to a height of about 30 feet, mercury which is 13·6 times denser than water, must rise to a height of about 27 inches. In 1643, he came [illegible]se an experiment which has immortalized his name.

Expt. 4. Take a thick-walled, long glass tube, about a metre long and a centemetre in diameter, and closed at one end. Fill it with clean, *dry* mercury, care being taken *to expel all traces of air* from the tube (fig. 170). For this purpose, close the open end of the tube with the thumb leaving a small quantity of air above the mercury: then incline the tube gently [illegible] allow the air bubbles to pass from end to end, thus in [illegible]n it the small bubbles of air that may adhere to the glass.

Now fill the tube *completely*, and close the open end with the thumb so as not to allow any air bubble between it and the mercury. Holding the tube firmly, invert it and immerse the open end in a small cistern of mercury (fig. 170, ii). On removing the thumb, mercury will be seen to descend in the tube, and after a few oscillations it comes to be stationary at a height which is generally about 78 cms. or nearly 30 inches* above the surface of mercury in the cistern.



(i) (ii)
FIG. 170
Torricelli's Experiments

We now proceed to find the explanation of the support of the mercury column in the tube, T. The clear space above the surface of mercury in T, is devoid of air, and hence is a vacuum,† usually called the Torricellian Vacuum. Hence the pressure at any point A, *inside the tube*, at the level of the surface of mercury in the cistern is that due to the column AB of mercury. This must equal the pressure at a point on the surface of mercury in the cistern *i.e.*, at the same level as A ; and this latter pressure is due to the atmosphere. It follows that the atmospheric pressure at the surface of mercury

* 76 cm = 29·992 in.

† It is not a complete vacuum, for it contains mercury vapour, the pressure of which, however, at ordinary temperatures, is practically inappreciable.

outside the tube is equal to the pressure due to the column of mercury, AB, standing in the tube ; in other words, the downward pressure of the atmosphere on the mercury surface in the cistern maintains the vertical column of mercury in the tube.

The simple apparatus of Torricelli's experiment constitutes a simple form of a BAROMETER, an instrument for measuring the atmospheric pressure.

The following experiments add further evidence of the fact that it is the atmospheric pressure which supports the vertical column of mercury in the tube.

Expt. 55. Fit up a barometer as indicated before and fix the tube vertically by means of a suitable stand. Note the

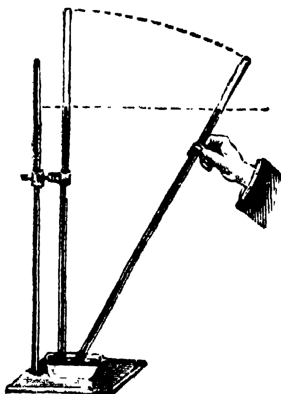


Fig. 171
Barometric height

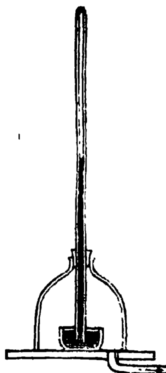


Fig. 172
Fall of mercury with reduction of air-pressure

height of the mercury in the tube above that in the reservoir. Then incline the tube at various angles to the vertical (fig. 171) and measure the vertical height in each case. It will be found that the *vertical height* of the top of the column above the mercury in the reservoir is always the same. This shows that since the atmospheric pressure is constant, the vertical height of the column of mercury supported by it must also be constant whatever be the inclination of the tube.

Expt. 56. Arrange a barometer so that the trough is under the receiver of an air-pump (fig. 172), the tube passing through a tightly fitting rubber stopper in the bell-jar. Exhaust the air gradually. As the pressure on the mercury in the trough is reduced, the level of the mercury column falls. On re-admitting the air into the bell-jar, mercury rises to the original level in the tube.

161. Pascal's Experiment.—Torricelli observed that the cause of the pressure of the air must be the same as the cause of the pressure of liquids *viz.*, the weight of the fluid itself; but he died in 1647, before he had the opportunity of submitting the problem to a test. This test was undertaken by Pascal. He argued that since the pressure in a liquid diminishes on going up towards its surface, the atmospheric pressure also ought to diminish on passing from the sea-level to a mountain-top. He requested his brother-in-law Perrier who lived near Puy de Dome, a mountain in the south of France, to try Torricelli's experiment. In 1648, two observations were made, one at the foot of the Puy de Dome, and the other at its top, a height of about 3565 ft.: the mercury column stood at 28 inches at the bottom and 24·7 inches at the top.

162. Value of the Atmospheric Pressure.—It has been proved in the previous article that it is the atmospheric pressure which supports the column of mercury standing within the barometer tube. Hence it follows that the hydrostatic pressure exerted by this column of mercury is a measure of the atmospheric pressure. If h is the height of this column, ρ is the density of mercury, then the atmospheric pressure will obviously be given by

$$\pi = h \rho g$$

The pressure of the atmosphere is not always constant but varies within certain limits round a mean or standard value. The **standard barometric height** is taken as 76 cms. and the pressure exerted

by a column of mercury of this height at sea level in latitude 45° at 0°C is called the **standard atmospheric pressure**. The density of mercury at 0°C is 13·596 gms per c.c. and the value of g in latitude 45° is 980·6 dynes per sq. cm. Hence in C.G.S. units, the standard atmospheric pressure

$$= 76 \times 13·596 \times 980·6$$
$$= 1{,}013{,}250 \text{ dynes/cm.}$$

It must be noted that the pressure exerted by a column of mercury of a given height will be different at different latitudes and at different temperatures. It is for this reason that the temperature and latitude should be clearly specified along with the height of the column to make the pressure definite.

In the F.P.S. system the value of the standard atmospheric pressure is given by

$$\pi = 1{,}013{,}250 \times \frac{(2·54)^2}{4·45 \times 10^5}$$
$$= 14·7 \text{ lbs. wt. per sq. inch}$$

for 1 inch = 2·54 cms

and 1 lb. wt = $4·45 \times 10^5$ dynes

On the English system of units the standard barometric height is sometimes taken as 30 inches. The standard atmospheric pressure in the F.P.S. system is then given by

$$\pi = 30 \times ·49 \times 32·2 \text{ poundals per sq. inch}$$
$$= 30 \times ·49 \text{ lbs. wt. per sq. inch}$$
$$= 14·75 \text{ lbs. wt. per sq. inch}$$

the density of mercury being ·49 lbs. per cu. in.

Thus the value of the normal atmospheric pressure may be *roughly* taken as one million dynes per sq. cm. or 15 lbs. wt. per sq. inch.

In recent times and specially in meteorology the atmospheric pressure is expressed in terms of a unit

called the **bar** which is equal to the pressure of one million dynes per sq. cm. Thus

1 bar = 10^6 dynes per sq. cm.

1 millibar = 10^3 dynes per sq. cm.

and the standard atmospheric pressure is approximately equal to 1013 millibars. The advantage of expressing the pressure in bars or millibars lies in the fact that the temperature or latitude need not be specified as is the case if the pressure is expressed in terms of the height of a mercury column.

If a gas or a liquid acts in such a manner as to exert a pressure of 15 lbs. per square inch of a surface exposed to it, the pressure is often spoken of as that of ONE ATMOSPHERE.

Since in the above calculation the pressure per unit area only has to be considered, the height of the column AB in the barometric tube is quite independent of the form and area of the cross-section of the tube, as well as the extent of surface of the mercury in the cistern.

It follows from the calculations given above that a square foot of surface is subjected to a pressure of 15×144 or 2,160 lbs. wt. or nearly a tons wt. Now the surface area of the body of a man of middle size is about 16 sq. ft. ; hence the pressure on his body amounts to an enormous pressure of 37,500 lbs. or upwards of 16 tons. We do not, however, feel this pressure, because at every point it is exerted equally in all directions. At the same time, it is evident that the body, being subject to a normal pressure at all points in its surface, is compressed to an extent which depends upon the elasticity of volume of its component parts ; the solid parts of the skeleton can resist a far greater pressure ; as regards the liquids in the organs and vessels, they are virtually incompressible ; the internal air, too having the same

density as the outside air, is under the same pressure. The compressing effect of the air pressure on the tissues of the body is one of the conditions to which the structure of the body is specially adapted, and we are not sensible of the effects, because we are always subject to them.

In balloon ascents, and on very high mountains, travellers experience a strong pressure of blood towards the nose and eyes, owing to the fact that the pressure of the enclosed air preponderates over the greatly diminsihed pressure of the surrounding outer air.

163. Barometers.—By fixing the Torricellian tube (art. 160) in a permanent position, we obtain a means of measuring the amount of the atmospheric pressure at any moment : and this pressure is measured by the weight of the column of mercury which it supports. Such an instrument is called a **Barometer.**

In constructing a mercurial barometer the mercury used must be pure and clean. Further, to drive out the air and moisture, the mercury must be carefully boiled in the glass tube. Ordinary mercurial barometers are either of the *cistern* or the *siphon* type. The barometer may be filled with any liquid other than mercury, for instance water, glycerine, etc. : but in that case, the tube would be inconveniently long, as the liquid used is less dense than mercury. In the *Aneroid* barometer no liquid is used ; it is not so accurate as a mercurial barometer, but is light and portable.

Cistern Barometer.—The Torricellian tube standing on its basin of mercury, is a **cistern barometer**. The atmospheric pressure is given by the height of the mercurial column measured always from the surface of the mercury in the cistern to the top of the mercury in the tube. If the pressure increases, the

mercury rises in the tube and the mercury surface in the cistern is lowered down. If the pressure decreases the mercury sinks in the tube and the mercury surface in the cistern rises up. Hence with the variation of the atmospheric pressure the mercury level in the cistern also varies. If, therefore, a scale be permanently fixed by the side of the tube, the zero mark of which is meant to be at the level of the mercury in the cistern, an error creeps into the readings called the **capacity error**, due to the lack of coincidence of the zero of the scale with the mercury level in the cistern under varying atmospheric pressures.

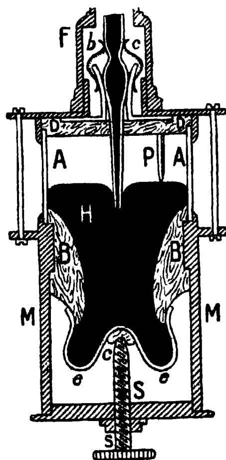


Fig 173
The cistern of a
Fortin barometer

By making the area of the tube small in comparison with that of the cistern, the rise and fall of mercury level in the latter can be made small; still for accurate work, it is necessary to make up for this defect by suitable devices.

Fortin Barometer.—The most convenient form of a mercury barometer for general use in accurate work is **Fortin Barometer**, shown in fig. 174. It is an improvement upon an ordinary cistern barometer in as much as an arrangement is herein made, so that the zero mark of the scale may readily be brought to coincide with the surface of the mercury in the cistern.

The usual arrangement of the cistern and the lower part of the tube of the

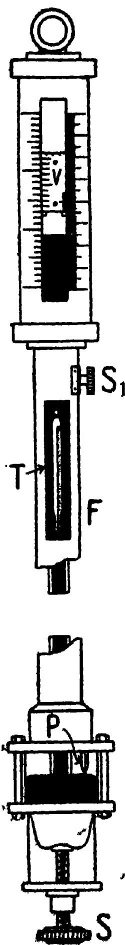


Fig. 174

instrument is shown in fig. 173. The cistern consists of a glass cylinder, A, which allows the mercury to be seen ; the bottom of the cylinder is cemented to a boxwood cylinder, BB, to which is fixed a chamois leather sack, *l*, forming the base of the cistern. The leather is provided with a small wooden button C, against which the screw S works, thus lowering or raising it as desired. The cistern is enclosed in an outer metal case M in the way shown in figure allowing the surface of the mercury to be seen through the glass cylinder. The screw S works through the bottom of this outer case. Fixed to the lid of the cistern is a small ivory pin P ; the point of this pin marks the zero of the scale on which the height of the barometric column is read.

The barometer tube drawn at the open end fits through a boxwood collar D in the disc which covers the cistern ; the cistern is closed in the upper part by a buckskin *bc* tied to the tube and a tubular fixed to the cover; this strip of leather prevents the escape of mercury from the cistern but transmits the atomospheric pressure by allowing free access to the air through its pores into the cistern.

The tube of the instrument is encased in a long brass tube F. (Fig. 174). At the top of this case, there are two longitudinal slits

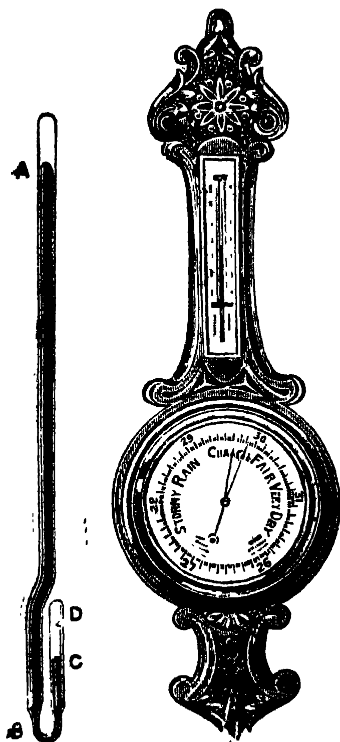
about 20 cms. long, cut parallel to the length of the tube and diametrically opposite to each other, so that the level of the mercury in the barometric tube may be seen. The scale to read the height is engraved on the outer tube along the edge of the front slit; it is graduated either in millimetres or in inches or both. The zero of this scale coincides with the tip of P. A sliding vernier V which can be moved up and down in the rectangular slit by a rack and pinion movement worked by a screw S_1, reads the height of the mercury column accurately when its lower surface is made to coincide with the mercury surface in the tube. At the lower part of the case is affixed a thermometer T, to indicate the temperature.

To read the barometer, the screw is moved up or down until the level of the mercury in the cistern just comes to touch the tip of the ivory pin. The vernier is next adjusted, until the top of the mercury column and the lower edge of the vernier appear in the same line. The reading is then taken off the vernier and is further subjected to certain *corrections* or *reductions* which are important for an accurate determination of the atmospheric pressure.

Siphon Barometer.—This is more convenient and portable than the cistern barometer. It has no cistern, but consists of a long glass tube, the open end of which is bent upwards (Fig. 175), so that the short open limb BD takes the place of the cistern. The long leg AB which is closed at the top, is filled with mercury, as in the cistern barometer. The difference of the mercury levels at A and C, in the closed and open limbs respectively of the tube, measures the height of the barometer.

To protect the mercury surface at C, the end of the shorter arm may be closed leaving only a pin-hole D at the side through which the communication is kept with the atmosphere.

Weather-glass or Wheel Barometer—The ordinary weather-glass or house-hold barometer is a form of siphon barometer. In the shorter leg, there is a float which rises or falls with the mercury. The



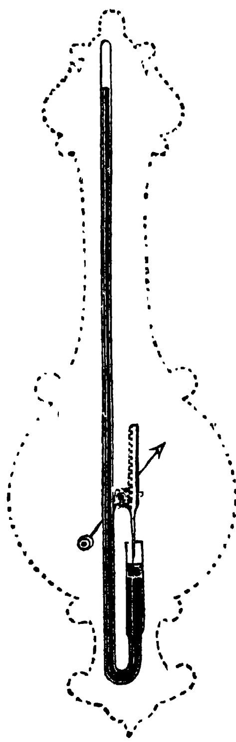


Fig. 175. Siphon Barometer

Fig. 176. weather-glass

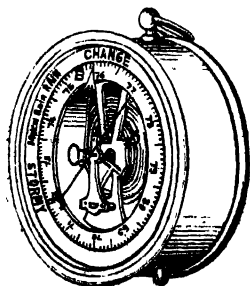
Fig. 177 wheel Barometer

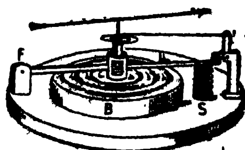
float is connected by a rack-and-pinion arrangement to the central wheel, to the axis of which is fixed a needle moving on a dial (Fig. 177). The dial is

mounted in the front of the tube so as to conceal its presence. It is graduated and marked *stormy*, *rain*, *variable*, *fair* etc. (Fig. 176). When the pressure varies, the float rises or sinks, and moves the index needle to the corresponding points on the scale. The rack-and-pinion arrangement is sometimes replaced by a pulley and a string carrying a counter-poise at the other end. The wheel barometer is a very old invention and was introduced by Hooke in 1683.

The weather-glass is neither very delicate nor very precise in its indications. Further, the indications on the dial as to the state of the weather are strictly of any value for the place for which the barometer is made, and differ for places at different levels and different latitudes.

Glycerine Barometer.—The vapour of pure glycerine has a very low pressure at ordinary temperatures and hence there is not much objection to the use of glycerine for barometric purposes as there would be to that of water. The column of glycerine (sp. gr. 1·28) corresponding to 30 inches of mercury (sp. gr, 13·59) is 318·5 inches or about 27 ft. so that the fluctuations of the barometer are magnified about 10·7 times. As, however, glycerine readily absorbs moisture from the air, it is usual to cover the liquid in the cistern with a layer of paraffin oil.





(*a*) Fig. 178.—Aneroid Barometer (*b*)

The Aneroid Barometer.—This form of barometer

is commonly used by geological and surveying parties, and also in air-crafts as it contains no liquid (from Gr. *a*, not, and Gr. *neros*, moist) and is hence more convenient to carry.

It consists essentially of a small chamber in the form of a shallow cylindrical box B (Fig. 178). The box is partially exhausted and closed with a thin metallic diaphragm which is corrugated. Variations in the atmospheric pressure cause the diaphragm to yield to an amount proportional to the change of pressure. In some forms, the chamber takes the shape of a thin-walled metallic tube in the form of a crescent which is closed and exhausted ; the ends of the tube separate or approach as the external pressure diminishes or increases. The motion of the diaphragm or the ends of the tube, as the case may be, is multiplied by a delicate system of levers and transmitted to an index moving over a dial whose readings are made to correspond to the readings of a mercury barometer.

The aneroid barometer is lighter, portable and less liable to injury than a mercurial barometer : it is sometimes made as small as a watch. Changes in temperature may produce some alterations in the readings which should for this reason be checked by occasional comparison with a standard mercurial baremeter.

Aneroid barometers are often used to measure the heights of mountains. Such an instrument is provided with a circular scale on the dial concentric with the scale of pressures and graduated directly in feet. This scale can be rotated round the centre of the dial and is so adjusted at the foot of a mountain of which the height is to be determined that the pointer reads zero. Then as the mountain is ascended the atmospheric pressure gradually diminishes and the pointer registers the actual height climbed. Evidently the readings are true only if the atmospheric pressure at the foot of the mountain remains constant during the ascent.

An aneroid barometer can be arranged to record its indications on a piece of squared paper by means of a pencil fitted to a long lever ; the paper is wound

on a cylindrical drum rotated by a clock-work. The whole arrangement is then called a BAROGRAPH or a self-registering barometer.

164. Variations in the Atmospheric Pressure and Weather.—When the barometer at a place is observed for several days, its height is found to vary during the same day as well as from day to day. The extent of these variations is from 78 cms. to 71 cms. or 31 in. to 28 in.

This shows that the atmospheric pressure is subject to variations. It is further observed that when the temperature rises, the barometer falls, and *vice versa.* The *daily variations* appear to result from the change of density of air, consequent on the expansions and contractions produced in the atmosphere by the variation of the intensity of radiation from the sun. Whenever there is a difference of temperature in any portion of the atmosphere and its neighbouring parts, currents in the air are produced : the air from the warmer region rising up and passing away through the upper regions of the atmosphere : thus the pressure in that particular portion is diminished and the barometer falls.

In the equator and between the tropics, the daily variations are rather regular ; the barometer sinks from midday till towards 4 P. M. and then rises and reaches its maximum at about 10 P. M. It then sinks again and reaches a second minimum towards 4 A. M. and a second maximum at about 10 A. M. Barometer changes are much greater and more rapid in the temperate zones than in the tropical regions.

The use of the height of the mercury column in a barometer,—rather of the changes in the height,—as to the prediction of the possible state of the weather is wide, as a change in the weather has been frequently found to coincide with a change in the atmospheric pressure. But the indications will differ

according to the meteorological conditions of the place. As a general rule, however, the following is interesting to note.

As dry air is heavier than damp air, (the density of water vapour being 0·6 of that of the air at the same temperature and pressure), the barometer rises ordinarily in dry air, indicating fair weather, and falls in moist air which generally precedes a rainy weather.

From the coincidence observed between the barometric heights and the state of the weather the following indications may be roughly remembered,—

HEIGHT	STATE OF THE WEATHER
79 cm ; 31 in	Very dry
78 cm ; $30\frac{2}{3}$ in	Steady
77 cm ; $30\frac{1}{3}$ In	Fair
76 cm ; 30 in	Variable
75 cm ; $29\frac{2}{3}$ in	Rain or wind
74 cm ; $29\frac{1}{3}$ in	Much rain
73 cm ; 29 in	Storm

Further, a rapid rise of the barometer on any day signifies a fine weather, but not lasting ; a slow movement or a steady height states the contrary. A sudden and rapid fall, on the other hand, indicates storm ; a slow continuous fall continuing for days together implies a lasting bad weather.

The Indian Government has Observatories in all the principal cities where, along with other observations, the barometric height is noted every day at 8 A. M., 10 A. M., and 4 P. M. The 8 A. M. observations are telegraphed at once to Simla, Calcutta, Bombay and Madras. At the central observatories maps are daily prepared and published. In these maps are drawn the lines, called **Isobars**, connecting places of equal barometric pressure and curves, called **Isothermal lines**, passing through places of equal

temperature. The strength and direction of the wind, and the state of the weather as well as of the sea are also entered herein. From these, a meteorological forecast of weather is issued daily.

165. Measurements of Heights by the Barometer.—It has been noted before in art 157 that the pressure of the air increases downwards to the surface of the earth just as the pressure increases with the depth of a liquid. But in a liquid which is almost incompressible, the pressure increases in proportion to the depth ; in air, however, it is not like that. As air is very compressible, a layer in the lower proportion of the atmosphere is compressed by the weight of the superincumbent layers : hence the pressure in the air increases much more rapidly than in proportion to the depth. So the law for the variation of the barometric height with the altitude is not a simple one. Very complete tables have, however, been prepared by which the difference in height between any two places may be readily ascertained, if the corresponding heights of the barometer be known. For small elevations we may roughly take that the barometric height falls *1 inch for the first 900 ft. ascended* (or 1 mm. for 10 metres), 1 inch for the next 1000 ft., 1 inch for the next 1100 ft., etc.

166. Height of the Homogeneous Atmosphere.—We have already seen that the pressure of the atmosphere diminishes as we rise up above the sea level. The exact law of variation of pressure with height is beyond the scope of this book. It is sufficient for our purpose to remember the variation referred to above. But one point of importance should be noted. Air, like all other gases, being highly compressible, its density varies directly in accordance with the pressure. It follows, therefore, that the density of atmospheric air diminishes with increasing height. From the law of variation of pres-

sure with height, the diminution of density with increase of height may be calculated. It may appear at first sight that the height at which this density is zero can then be found out from this law and hence the extent of the earth's atmosphere may be determined. But unfortunately we cannot obtain a finite height for which the density is zero, so that the actual extent of the atmosphere cannot be definitely known. We may, however, infer from observations made by sounding balloons and also from other sources of information that the amount of air present at a height of even 50 miles above the surface of the earth is not inappreciable.

If, however, we *assume* that the density of air in the atmosphere is the same throughout and is equal to that at the surface of the earth it is possible to find out a height H above which there is no air at all. The upper surface of such an imaginary atmosphere will then correspond to that of a sheet of water. For, if ρ be the density of air at the earth's surface and g the acceleration due to gravity,

$$\rho g H = \pi$$

where π is the atmospheric pressure

$$\text{Hence } H = \frac{\pi}{\rho g} = \frac{1{,}013{,}250}{\cdot 001293 \times 981} \text{ cm.}$$

$$= 8 \text{ km. approximately}$$

This height H is known as the **height of the homogeneous atmosphere.** It is the height of a hypothetical atmosphere of constant density which would give exactly the same pressure and hence the same density as the real atmosphere at the surface of the earth. To express it in another way : the height of the homogeneous atmosphere is the height to which the real atmosphere would extend had it been compressed into a homogeneous

mass of air having the same density as that at the surface of the earth.

ARCHIMEDES' PRINCIPLE APPLIED TO GASES

167. Buoyancy of the Air.—As the air, like liquids, exerts pressure equally in all directions, Archimedes' Principle applies in this case too. In other words, bodies immersed in air or any other gas, are buoyed up, as in liquids, by a pressure equal to the weight of the air or the gas displaced. The loss of weight of a body in the air is demonstrated by means of the **Baroscope** shown in fig. 179. It consists of a scale beam, at one end of which a hollow glass sphere is supported, and at the other a lead counterpoise ; the latter arm is also provided with a rider screw for the final adjustment.

Expt. 57. Adjust the rider screw until the beam is horizontal. Place the whole under the receiver of an air-pump and exhaust the air inside. The sphere sinks showing that its weight has apparently increased (fig. 179). Restore the former state to the beam by re-admitting the air.

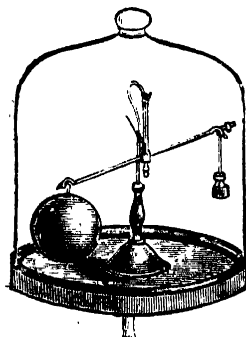


Fig. 179
Baroscope

When in air, the sphere as well as its counter poise, are buoyed up by the weight of the air displaced. But as the sphere has a larger volume, it displaces a larger volume of air and is consequently acted upon by a larger buoyant force in air. Though the two are balancing each other in the air, it is evident that the true weight of the sphere is

greater than that of the counterpoise. On removing the air from inside the apparatus, the buoyant forces disappear and the true weights become evident.

It is thus evident that when a body is weighed in air, the weight obtained may be called its *apparent weight* and is less than its true weight in vacuo. In determining the weight of an object precisely, a correction for the buoyancy in the air must be allowed for both the body to be weighed and the weights used.

It follows from the Principle of Archimedes that if the weight of a body is less than that of the air displaced by it, in other words,—if the body be lighter than the air,—the body will be buoyed up and will rise in the atmosphere until it reaches a layer of the same density as its own ; the force causing the ascent being the excess of the buoyancy over the weight of the body. This is the reason why smoke, vapour, a fire-balloon and air-balloons rise in the air.

168. Balloons.—The buoyancy of the air is utilized for an important purpose *viz.*, the ascent of a balloon in the air. A **Fire-Balloon** consists of a paper envelope with a wide opening below, in the centre of which is a piece of sponge or cotton wick, held in a wire-frame, and soaked in methylated spirit. The sponge is ignited and the bag is filled with the heated air which, being lighter than the cold air, causes the balloon to rise.

A **balloon** is essentially an air-tight envelope or a bag of silk or some other light material filled with some gaseous substance like helium, hydrogen, coal-gas, etc., which is lighter, bulk for bulk, than the air at the surface of the earth and serves to float the apparatus in the air (fig. 180). In the usual form it is spherical with a light car or basket suspended below

it to carry passengers. The car is suspended by cords attached to a net-work covering the upper half of the balloon. The necessary condition of the ascent is that the weight of the air displaced must be greater than that of the balloon and its load.

The difference between the weight of a balloon and that of the air displaced by it, is called the **lifting power** of the balloon. If the approximate weight of 1 cubic metre of air is 1300 gms. that of hydrogen, 89 gms., that of coal gas., 750 gms., that of air heated to 200°C, 750 gms., then the lifting power per cubic metre of hydrogen is 1211 gms. ; of coal-gas, 550 gms. ; of heated air, 550 gms. Though coal-gas has a lifting-power much smaller than hydrogen, yet it is generally employed on account of its cheapness, and of the facility with which it can be procured.

Balloons are not fully inflated at the commencement of the ascent, for as the balloon rises, the density of the air diminishes and the external atmospheric pressure on the balloon continually diminishes. In consequence, the gas inside it expands in the same ratio as the pressure diminishes outside (see Boyle's Law) till the ballon is fully distended. Up to this time the lifting power remains nearly constant. Suppose, for instance, the atmospheric pressure has diminished to one-half its value, the volume of the balloon will then be doubled ; it will then displace a volume of air twice as great as before, but only of half the density, so that the buoyancy will remain the same. This conclusion, however, is not quite exact, as the solid parts of the balloon do not expand like a gas.

If a balloon continues to rise even when it is fully distended, its lifting-power diminishes rapidly, for the volume of the displaced air remains the same, but its densitydiminishes. A time, however, arrives

when the weight of the air displaced is equal to that of the balloon itself; the balloon can rise no more and comes to rest in the region after a few oscillations, only to be drifted by the current in the air.

At the top of the balloon there is an opening, closed by a valve held to a spring, which the aeronaut can open at pleasure by means of a cord.

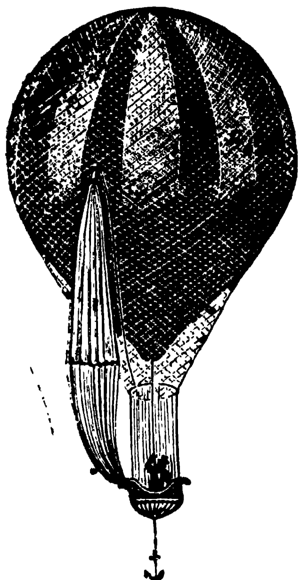


Fig. 180.—Balloon

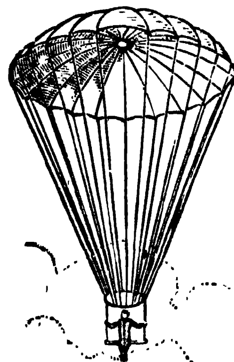


Fig. 181—Parachute

When the aeronaut wishes to descend, he opens this valve by means of the cord, thus allowing the gas to escape. To rise again or to make the descent less rapid, sand-bags kept as ballast in the car are gradually emptied.

The descent from a balloon in the mid-air is sometimes effected by means of a **parachute** (fig. 181). It is a huge umbrella-like contrivance, from the circumference of which hang cords supporting a

small car. The resistance of the air opens the parachute and acting upon its large surface moderates the rate of its descent. It has a hole in the top, which allows the air to escape slowly and thus the parachute is kept upright.

Almost from the very beginning of ballooning, some method of directing the balloon to a pre-determined goal had been sought by the inventors, for one of the strong ambitions of the human race is to fly and not simply to rise in the air, though the latter is undoubtedly a necessary step. To accomplish this, a machine propeller is needed to drive the machine after it has been lifted up in the air, and a suitable rudder to direct its course. Such a machinery will again have a weight and the gas-bag must be enlarged to counterbalance it. The whole constitutes the dirigible balloon or the **air-ship**.

An air-ship is a dirigible balloon, *i. e.*, it can be driven by means of machine **propellers** P, P (fig. 182)

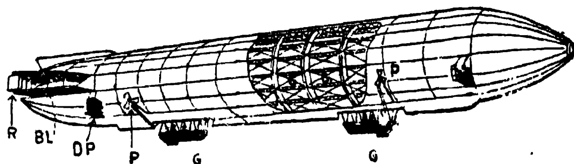


Fig. 182

A Zeppelin air-ship

after it has been lifted up in the air, directed in its course by a suitable **rudder** R. To support the weight of these machineries the *gas-bag* is enlarged. It is generally given an elongated cigar-like shape, so that the resistance of the air to its motion may be the least possible. As a matter of fact, several gas bags, separated from one another are put within suitable compartments which are encased in an outer envelope.

G. G are two **gondolas** or cars, each of which contains a motor which works one of the propellers which are rigidly connected to the body of the ship.

169. Flying Machines.—The term flying machine is applied to all forms of air-craft which are heavier than air, and which lift and sustain themselves in the air by mechanical means. In this respect they are distinguished from balloons which are lifted and sustained in the air by the lighter-than-air gas they contain.

Aeroplanes.—**Aeroplanes** are those forms of flying machines which depend for their support in the air upon extended metal surfaces which are called **sails** or **planes**. They are commonly driven by propellers actuated by motors. When not driven by power, they are called **gliders**.

Broadly speaking, an aeroplane (fig. 183) consists of :

(*a*) A metal body, more or less cigar-shaped and mounted on **skate-wheels**, S.

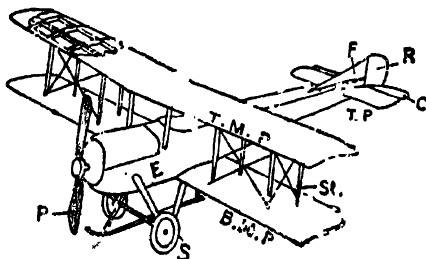


Fig. 183.
An Aeroplane

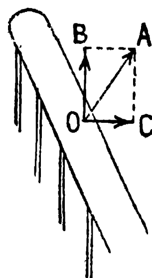


Fig. 184.

(*b*) **Main planes** T. M. P. and B. M. P.—These are large metal sheets fitted in front, which support the aeroplane in

the air ; the upper planes are bent from front to rear, with a convexity of surface upwards.

(c) A **propeller**, P—worked by the motor-engine and pulling the machine behind it through the air.

(d) The **tail**, which carries a vertical steering rudder R to direct the aeroplane. The theory of the lift and support of an aeroplane is that of an inclined plane driven in a more or less horizontal direction, so that the air impinges on the inclined under-surface. The wind pressure on the plane acts normally to the plane in a direction OA (fig. 184) and may be resolved into a vertical component OB and a horizontal component OC.

Just when leaving the ground, the speed of the plane must be sufficient to make OB slightly greater than the weight of the aeroplane. The component OC is to be overcome by the pull of the propeller. These forces are known as the 'lift' and 'drift' respectively. Hence, knowing the weight to be supported, the power necessary to keep it up can be calculated. It is obvious then that to stay up in the air, an aeroplane must move swiftly through it ; the heavier it is, the faster it must go.

Exercise XVIII

1. State in a general way the use of the barometer,

   (a) in measuring the heights of mountains.
   (b) in indicating the state of the weather.

2. A solid floats partly submerged in a liquid when the vessel which contains it is in air ; if the vessel be placed in a vacuum, will the solid sink, rise or remain stationary ?

3. Why does water gurgle out instead of issuing in a steady stream, when a bottle full of water is inverted ?

4. In selling diamonds by weight, which of the following is advantageous to the seller :—

(*a*) the barometer should be high or low ?

(*b*) the weights used should be of rock-crystal or of platinum ?

5. I litre of hydrogen and a litre of air weigh about 0·09 gramme and 1·3 grammes respectively at a certain temperature (t) and pressure (p). What will be the capacity of a balloon weighing 10 kilogrammes, which just floats when filled with hydrogen having the same pressure (p) and the same temperature (t) as the air ? [C. U.—1912]

6. Describe an experiment to prove that the air exerts pressure. How is this pressure measured ?

If a certain pressure is equal to that exerted by a column of mercury of height 760 mm., find its magnitude. (Sp. gr of mercury—13·6). [C. U.—1917]

7. Explain clearly what you understand by the atmospheric pressure.

Describe experiments to prove the existence of atmospheric pressure. How is it determined ? If it is equal to that of 32 inches of mercury, find its magnitude. [Sp. gr. of mercury =13·6]. [C. U.—1918]

8. Explain fully the meaning of the statement, 'The atmosphere exerts a pressure of 15 lbs. per sq. inch, nearly.'

How would you verify the statement experimentally ?
[C. U.—1919]

9. Describe any form of barometer you have used in your laboratory. Give the directions necessary for reading the atmospheric pressure. [C. U.—1921]

10. A glass tube, 20 inches long, closed at one end and entirely filled with mercury, is inverted over a mercury trough. State what happens, giving reasons. [C. U. 1922-23]

11. Describe an experiment to show that the principle of Archimedes can be applied to gases also. A flask is first weighed with its mouth open, then with its mouth well-corked by a rubber stopper ; what difference will you notice ?
[Pat. U—1919]

12. Express the normal pressure of air in absolute units. (Sp. gr. of mercury=13·6 ; g=981 dynes). [C. U.—1927]

13. Explain what you mean by the atmospheric pressure Give a brief description of any form of barometer.

Find the height of a glycerine barometer when the water barometer stands at 32 feet. (Sp. gr. of glycerine=1·27).

[C. U.—1928]

14. What is the effect of the pressure of the atmosphere on the weight of a body ? Give reasons for your answer, and describe an experiment by which this effect can be demonstrated.

[C.U.—1934]

CHAPTER XXI

BOYLE'S LAW

170. Expansion of Gases.—The three variables in the case of a gas are its volume, pressure and temperature. To specify a quantity of a gas by volume, the temperature which it possesses and the pressure to which it is subjected, must be mentioned.

A gas, unlike a solid or liquid, alters considerably in volume for small changes of pressure, even though the temperature remains constant. The relation between the pressure and the volume of a given mass of a gas *at a constant temperature* is found to conform generally to a definite law, called **boyle's law**. The law was discovered in 1662 by the HON. ROBERT BOYLE (1626—1691) in England, and also independently, in 1676, by MARIOTTE in France.

Similarly, the law that connects the rise of temperature and the increase of volume of a gas *under a constant pressure* was first enunciated in 1787 by CHARLES, a Frenchman, and is called *Charles' Law*.

171. Boyle's Law.—The law states the *volume of a given mass of a gas at constant temperature is inversely proportional to its pressure.* Thus if p be the pressure and v be the volume of a given mass of gas, then, according to the law, we have

v varies as $1/p$

or $v = k.\ 1/p$, where k is a constant.

or $pv = k$ ... ... **(99)**

Hence it follows that if a *fixed quantity* of a gas at a constant temperature has volumes denoted by

v_1, v_2, v_3, etc., under pressures denoted respectively by p_1, p_2, p_3, etc., then we must have

$$p_1v_1 = p_2v_2 = p_3v_3 = \text{etc.}$$

It will be seen that Boyle's law may be stated in terms of the pressure and density of the gas. For, if a given mass of a gas has a valume v_1 and density d_1 under a pressure p_1, and a volume v_2 and density d_2 under pressure p_2, than since

$$\frac{d_2}{d_1} = \frac{v_1}{v_2} \text{ and } \frac{v_1}{v_2} = \frac{p_2}{p_1}$$

we must have

$$\frac{d_1}{d_2} = \frac{p_1}{p_2} \qquad \ldots (100)$$

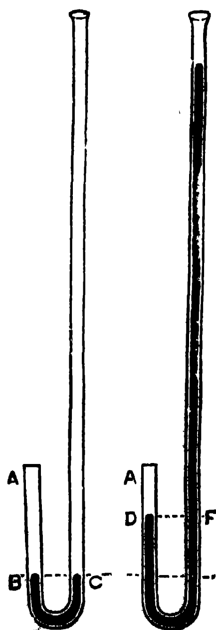
i.e., the density of a gas at a constant temperature is directly proportional to its pressure.

Experimental Verification of Boyle's Law—Boyle's law may be experimentally verified by means of a tube shown in fig. 185 similar to what was used by Boyle in his experiments to establish the law, and is generally called *Boyl's Tube*. It is simply a glass U-tube having one arm about 6 in. long and closed at the top, and the other arm, open, and about 36 inches long and is mounted on a vertical board. Both the limbs of the tube are usually graduated in the same way from a zero mark at the same horizontal level.

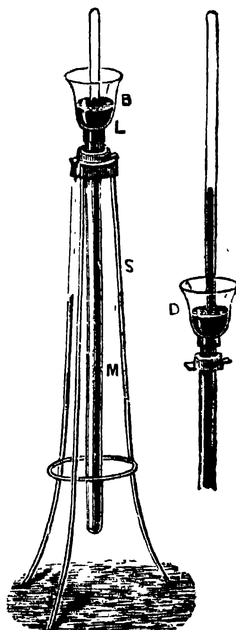
Expt. 58. Pour a small quantity of clean mercury into a Boyle's tube, and adjust by tilting the tube, so that the surface of the mercury in both the limbs is at the same level. Now the air enclosed in the closed arm AB is at the atmospheric pressure. To get its volume, read the scale AB ; it is assumed here that the bore of the tube is uniform, and the unit in which the volume is measured is evidently the capacity of the tube per uuit length.

Now pour mercury again into the open limb ; notice that

the volume of the air in the smaller limb is gradually reduced. Continue to do this until the volume of the contained air is half of what it was the atmospheric pressure; in other words, AD in fig. 185(a) is half of AB in fig. 185(b). Measure the height of the mercury column above DF. It will be found to be exactly equal to the height of the barometer at the time of the experiment. The pressure at F is, therefore, equal to that of two atmospheres which must also be the pressure of the air in AD.



(a) (b)
Fig. 185
Boyle's tube
Verification of Boyle's Law



(a) (b)
Fig. 186
Mariotte's apparatus

Thus the experiment proves that when the pressure of the air in the closed limb is *doubled*, the volume is *halved*.

Expt. 59. Arrange the apparatus as in the first part of Expt. 58 and note the volume of the air enclosed when it is at the atmospheric pressure.

Now pour mercury into the open limb, step by step, so as to raise the level about 3 inches at each step; and note the pressure and volume of the air in the closed limb at each step. The pressure at each step is given by the atmospheric pressure which is acting on the surface of mercury in the open limb, *plus* the pressure due to the difference of the mercury levels in the two limbs.

Arrange the readings in a tabular form.

It will be seen from these results that the value of the product of the volume of the air and the corresponding pressure is practically constant.

To demonstrate the truth of the law for pressures less than one atmosphere, a different arrangement known as **Mariotte's apparatus** is required. It simply consists of a graduated tube and a deep trough BLM to contain mercury, fixed on a suitable stand S [fig. 185 (*a*)].

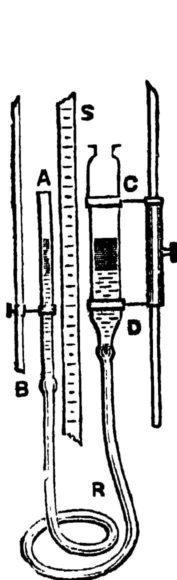
Expt. 60. Pour mercury into the graduated tube, until it is about two-thirds full, leaving the upper part to be occupied by the air. Place the thumb over the mouth of the tube, and invert it in the trough containing mercury [Fig. 185 (a)]. Lower the tube until the mercury inside and outside the tube is at the same level. Measure the length now occupied by the air which is at the atmospheric pressure.

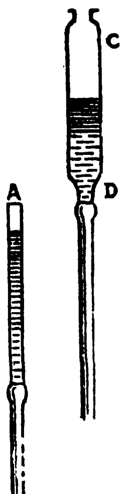
Raise the tube slowly; the mercury recedes from the closed end, showing that the air in it expands, owing to the decrease of pressure acting on it [Fig. 185(b)]. Measure again the length occupied by the air, and determine its pressure by subtracting the height of the column of mercury standing in the tube above the mercury surface in the trough from the barometric height observed from a Fortin barometer. As before, the tube is assumed to be uniform in cross-section; hence the volume of the air in the tube is proportional to the length occupied by it.

Verify that pv is almost constant.

The apparatus nowadays used in the laboratory for the above purpose is much more convenient than Boyle's tube. This consists of two glass tubes AB and CD connected by means of a long, thick-walled

India-rubber pressure tubing, and fitted by means of adjustable clamps on two vertical uprights (fig. 187).





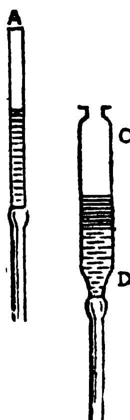


Fig. 187 Fig. 188 Fig. 189

Verification of Boyle's law

AB is uniform in cross-section and closed at the top and contains a certain quantity of *dry* air. The lower part of the glass tubes and the whole of the India-rubber tubing contain mercury. A wide scale runs along the vertical board in the middle. CD acts as a mercury reservoir. The pressure to which the gas in the closed limb is subjected can be varied by raising or lowering the mercury reservoir, thus allowing the law to be tested for pressures *both* greater and less than the atmospheric pressure.

Expt. 61. Fix the tube AB (fig. 187) at about the middle of the stand. Slide CD until the mercury stands at the same level within the two tubes. The pressure of the air enclosed is equal to the, atmospheric pressure, and its volume is proportional to the length of AB occupied by the air as read on the scale. Read the barometer to get the atmospheric pressure.

Now raise the tube CD gradually (Fig. 188) ; the volume of the enclosed air is reduced more and more. The pressure at each step is equal to the atmospheric pressure *plus* the pressure due to a vertical column of mercury standing between the levels in the two tubes. Note each time the volume of the air and the corresponding pressure with the help of the attached scale.

In the next series of operations lower CD below AB (Fig. 189). The pressure on the enclosed air is this time less than the atmospheric pressure by that due to a column of mercury standing between the levels of the two tubes. Take readings in several steps.

Record your results thus .

Height of barometer— cms. (H)

Difference of mercury levels, h	Total pressure of air $H \pm h = p$	Volume of air $= v$	Total pressure × volume $= pv$

Find the product of p and v and show that this is almost constant.

The relation between p and v may be conveniently expressed graphically : if a curve be plotted so that the abscissae represent the pressures and the ordinates the corresponding volumes of a given mass of gas at a constant temperature, the form of the curve is obtained as shown in fig. 190. The curve is of the form of what is called the Rectangular Hyperbola.

More refined experiments performed by Regnault and others

have shown that Boyle's law is not obeyed by gases at very high pressures. For ordinary pressures, however, the law does very nearly hold in the cases of so-called permanent gases *viz.*, oxygen, hydrogen, nitrogen, etc.

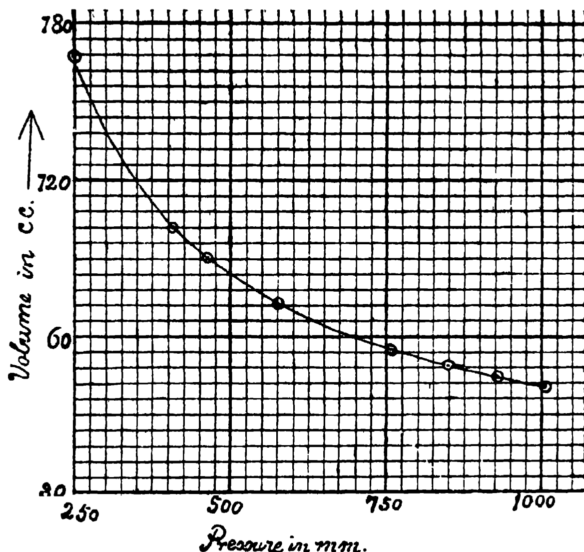


Fig. 190

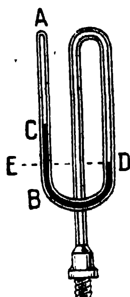
Verification of the volume of a gas with pressure

172. The Manometer.—The MANOMETER or the pressure-gauge is an instrument for measuring the pressure of a gas or of a vapour. The pressure is generally expressed in terms of the atmospheric pressure and is often measured by means of a column of mercury.

A Siphon Gauge consists of a glass tube bent to the form of a U, as ABD in fig. 191 and contains mercury at the bend. The end D is in communication

with the vessel, the pressure in which is to be measured.

When pressures which are not very considerable are to be measured, the end A of this gauge is often *open* to the air. Suppose that on connection with the vessel the mercury levels in the two limbs are at C and D. Now the pressure at D, equals the pressure at C (which is here the atmospheric pressure) *plus* the pressure of a column of mercury, the height of which is given by the difference of readings of C and D taken with the scale attached.



191 Fig.
A Siphongauge

The choice of a liquid to be used in the gauge will depend to some extent on the pressure to be measured: with a dense liquid like mercury, the 'head' of mercury *i.e.*, the difference of two levels in the two limbs would be small. If a liquid of smaller specific gravity be used, the 'head' necessary to measure a given pressure will be large in the inverse ratio of the specific gravities. Sulphuric acid (sp. gr., about 1·84) is often used; but it absorbs moisture very quickly and has its density altered thereby. Water may be employed in some cases but it evaporates rapidly; further, the pressure due to the water vapour may cause error.

For the measurement of *high* pressures, the end A of the gauge is closed as in fig. 191, so as to enclose a quantity of air. As the pressure on the mercury on the side D is increased, the mercury level in this limb is driven down, and the air in AC is compressed. The extent to which the air is thus compressed indicates the pressure to which it is exposed and this pressure is obtained by the application of Boyle's law. The pressure at C is due to the compressed air; in most cases, the pressure due to a column of the mercury of the height given by the difference of the levels of mercury in the two

limbs, is negligibly small compared with that due to the compressed air. In such cases, the pressure at D is roughly equal to that at C.

For measuring *low* pressures, the tube AB contains no air and is completely filled with mercury. So long as the end D is open, the atmospheric pressure forces the mercury up the arm AB to the top of the tube. When the pressure at D is sufficiently lowered, the mercury in AB falls. Suppose the levels are at C and D as in fig. 191 ; the pressure above C is zero ; draw DE horizontal through D. Pressure at D equals that at E which again is mesured by the height of the column EC. This form of gauge is commonly used with an air-pump.

Exercise XIX

1. The volume of an air-bubble increases six-fold in rising from the bottom of a lake. Find the depth of the lake. (The barometer reading=70 cms ; and sp. gr. of mercury=13·6).

2. A U-tube open at one end and closed at the other, is partially filled with mercury (density=13·6). The closed end of the tube contains some air, and the mercury in the open limb stands 30 cms. higher than it does in the closed limb Find in C. G. S. units the intensity of pressure on the air in the closed end of the tube. [C. U.—1910]

3. A faulty barometer contains some air which occupies 10 c. c. if it stand at 740 mm., when a true barometer indicates a pressure of 750 mm., find the volume the air will occupy at the standard pressure 760 mm. [C. U.—1911]

4. What volume does a gramme of hydrogen occupy at 0 C., when the height of the mercurial barometer is 750 in millimeters ? [1 c. c. of hydrogen weighs 000008958 grammes at 0°C and 760 millimeters.] [C. U.—1913]

5. A litre of air weighs 1.293 grammes at a pressure of 76 cm. and temp. 0°C. What will be the weight of a litre of air at the same temperature, when the barometer stands at 78 cms. ? [C. U.—1915]

6. A given quantity of a gas is allowed to expand to 1·5 times its original volume. What will be the pressure it will exert, if it were originally at a pressure of 750 millimeters of mercury, the temperature remaining constant throughout ?

Describe an experimental arrangement by which your result may be verified. [C. U.—1916]

7. What is the pressure of a gas in a closed space due to it? Explain how the relation between the volume of a given quantity of gas and pressure may be determined experimentally. [Pat. U.—1916]

8. A volume of air at the standard temperature and pressure is compressed to $\frac{1}{5}$th of its original volume. What will be the new pressure?

9. A narrow glass tube open at both ends, is partially dipped in a vessel containing water. The upper end is closed by the thumb and the tube taken out of water. State what will happen and why? [C. U.—1922]

10. State Boyle's Law. How may it be experimentally verified for pressures greater and less than the atmospheric pressure? [C. U.—1921 : '21 ; '24 ; '27 ; '31]

11. An accurate barometer reads 30 in. when one containing air above the mercury reads 24 in. If the tube of the latter be raised 3 in., the reading becomes 25 in. Find what length of the tube the air would occupy if brought to the atmospheric pressure. [C. U.—1924]

12. A tube six feet in length closed at one end is half filled with mercury and is then inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of the mercury inside the tube? [C. U.—1931]

13. Briefly describe a mercury barometer and explain its principle.

The height of a barometer is 75. cm. of mercury, and the evacuated space over the mercury surface has a volume of 10 c.c. 1 c.c. of air at the atmospheric pressure is introduced into the evacuated space. What is the new reading of the barometers? (The cross section of the tube is unity.) [C. U.=1929]

14. A borometer whose cross-sectional area is 1 sq. cm. has a little air in the space above the mercury. If it is found to read 71·8 cm. when the true height is 77·0 cm., determine the volume of the air present in the tube, measured under normal conditions. [C. U.—1937]

15. Slate Boyle's law, and describe an experimental

arrangement for verifying it for pressures less than one atmosphere.

Express the normal pressure of air in absolute units (Sp. Gr. of mercury=13·6 and $g=981$ cm. per sec. per sec.)

The density of air of N. T. P. is 0·00, 129 gm per c.c. find the alteration in the weight of 15 litres of air when the barometer falls from 76 cms. to 74 cms. [C. U.—1938]

CHAPTRE XXII

HYDROSTATIC MACHINES—PUMPS

In the present chapter we shall describe some of the more important forms of hydrostatic appliances which depend for their action upon the properties of liquids and gases.

173. The Pipette.—The **pipette** (fig. 192) consists of a glass tube with a bulb blown on to it about half way down. It is open at both ends and terminates below in a small tapering mouth. It is used for removing a liquid from one vessel to another. Water is introduced in the tube either by suction or by direct immersion. If the upper end be now closed with the finger and the tube be then withdrawn water will not fall, being acted upon, from outside by the atmospheric pressure which becomes equal to the total pressure due to the air and water enclosed inside. The lower end is then placed in a vessel to which the liquid is to be transferred. On re-opening the upper end the atmospheric pressure is allowed to act in the inside also and the enclosed water flows out. This flow can, if desired, be stopped by closing the upper end again.



Fig. 192
Pipette

174. The Siphon.—The **siphon** is an instrument by means of which a vessel filled with a liquid may be emptied, when the ordinary process of pouring the liquid off is not convenient or desired. It

is a bent tube of the form shown in fig. 193 open at both ends, one limb being longer than the other. First

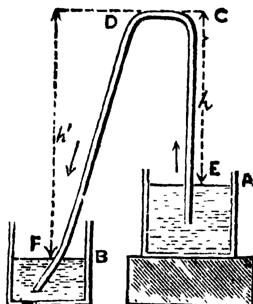


Fig. 193
The siphon

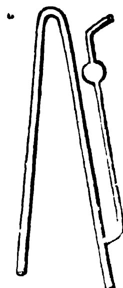


Fig. 194
The aspirating siphon

of all, it is filled with the liquid to be drawn off; the ends are then temporarily closed with the fingers, and the shorter leg is placed below the level of the liquid in A, the vessel to be emptied The other end is outside this vessel and below the level of the liquid surface in A. On opening the two ends, the liquid begins to flow out at once through the longer tube.

The principle of action of the instrument is readily understood from the following considerations. Let CD be the highest portion of the tube, which is horizontal, and let h and h' denote the *vertical* heights of the points C and D above the liquid levels at A and B respectively. Then

$$\text{Pressure at C} = \pi - hdg$$

$$\text{Pressure at D} = \pi - h'dg$$

where π is the atmospheric pressure and d, the density of the liquid.

Now as $h < h'$

$\therefore$ pressure at C $>$ pressure at D.

Hence water will flow from C towards D, and the atmospheric pressure which is greater than the

pressure at C will again raise water to C ; thus a continuous flow will be maintained.

The two conditions which must hold, so that the siphon can act are—

(1) The level A of the liquid in the vessel which is to be emptied, must be above the level B in the other vessel.

(2) The height of the top of the siphon above the liquid in the vessel to be emptied must be less than the height of the corresponding liquid barometer.

For convenience in filling, the siphon is often made in the form, called the ASPIRATING SIPHON, shown in fig. 194 which is provided with a side-tube. One end of the siphon is inserted in the liquid to be removed, while the other end is closed, and the operator applies suction at the side-tube till the liquid flows over. In siphons for commercial purposes, the suction is usually produced by a pump.

In TANTALUS' CUP (Fig. 195) a siphon is con-

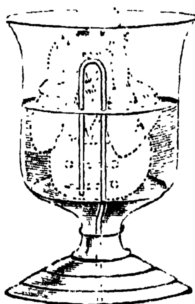


Fig. 195
Tantalus' Cup

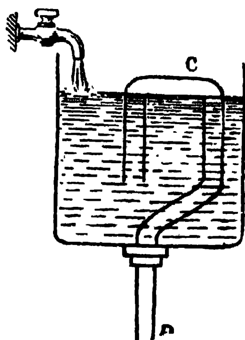


Fig. 196
Automatic flushing Tank

cealed inside a figure representing Tantalus, placed

in the vase, whose mouth is just above the top of the siphon. Water is poured into the vessel, and no sooner does it reach the top of the siphon and approach the level of his lips than the water begins to flow away through the siphon, purporting to leave him as thirsty as ever.

The siphon may be employed to produce an intermittent flow of a liquid. Fig. 196 represents a vessel in which there is a siphon with its shorter arm terminating near the bottom, while the longer arm passes through the bottom. If a small stream of water flows into the vessel, the level will gradually rise both in the vessel and in the shorter branch of the siphon, till it reaches the top of the bend, when the tube is filled which the liquid. The siphon then acts, and powerful rush of water issues through the pipe C until the vessel is emptied up to the mouth of the shorter arm of the siphon. If the supply of liquid be allowed to continue, this siphon will recommence its action when the level of the liquid again rises to the level of the bend. The same principle is applied in the *automatic flushing tanks* used for sanitary purposes.

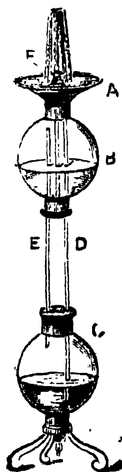


Fig. 197
Hiero's fountain

175. Hiero's Fountain.—It derives its name from its inventor Hiero, who lived at Alexandria in 123 B. C. It consists of a dish A and two globes B and C. A tube D runs from A to the bottom of the lower globe C (fig. 197), a second tube E connects the upper parts of the two globes and a third tube F proceeding from the bottom of the upper globe B passes through a cork in the centre of the dish A and ends in a fine jet.

The upper globe B is filled with water by remov-

ing the cork fitted to the dish A. The cork is then placed in position and water is poured on A. The water flows through D to collect in the globe C and displaces the air in it. This in its turn passes up into the globe B through the tube E and out through E in a jet as represented in the figure.

176. The Syringe.—This instrument is the simplest form of the pump for raising water. It consists of a hollow cylinder AB (fig. 198), terminating at its lower end in a nozzle C and provided with a solid, air-tight piston working inside it.

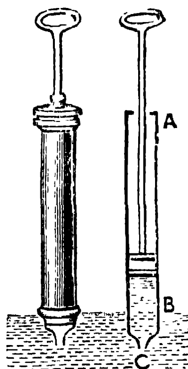


Fig. 198
Syringe

Expt. 62. Place the nozzle C of the syringe, with the piston at the bottom of AB, under the surface of water. Raise the piston ; the pressure of the air, acting on the upper surface of the liquid, forces it into the cylinder to fill the vacuum which would otherwise be formed below the piston. Take out the syringe when sufficient liquid has been drawn up. The liquid may be ejected again through the nozzle C by reversing the motion of the piston.

The principle of the syringe and the various forms of pumps is that of suction. This consists in enlarging the volume of a space to which the liquid has access ; the pressure within the space is thus reduced and the atmospheric pressure forces the liquid into the space to fill up the partial vacuum. This principle was not understood by the ancient philosophers who tried to explain the rise of the liquid by saying that *Nature abhors a Vacuum.* In inhalation, the muscles of the chest cause the lungs to expand, thereby reducing the internal pressure ; so the air is driven in. The act of drinking water is similarly explained.

177. Valves.—Valves are used in most of the hydrostatic machines. They are made so as to yield to an excess of pressure on one side only; an excess of pressure on the opposite side will close the valves. They thus allow passage to water, air etc., through the holes they close, in one direction but not in the other.

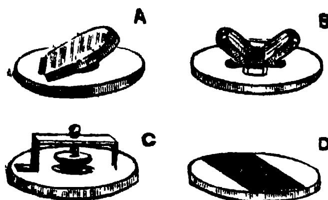


Fig. 199
Valves

Fig. 199, A represents a *hanging flap* valve, it is a flat disc turning about a hinge in its upper edge and thus opening or closing the passage over which it is fitted. In the ordinary bellows (fig. 200) the valve is a leather flap K. This is raised when the bellows are being expanded and allows the air to enter; when the bellows are compressed, the flap is pressed down tightly on the hole which is thus closed, and the air is forced out through the nozzle.

Fig. 199 B represents a *double flap* valve, and fig. 199 C, a *conical* valve. When a fluid is forced

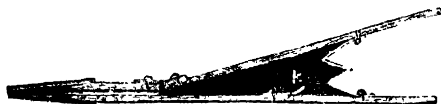


Fig. 200
The ordinary bellows

through the valve from a downward direction, the cone is raised and the fluid passes upwards. The pressure in the opposite direction serves only to drive the cone more closely against the hole over which

it is fitted. The cone is prevented from moving far from the orifice by a suitable guide.

A form of valve used in many air-pumps is shown in fig. 199, D. It consists of a strip of oiled silk, secured firmly at both ends to a plate of brass over a narrow slit in the plate, meant for the passage of the air. When the air is forced against the valve through the orifice, the silk is lifted slightly and the air escapes; if the air is forced in the other direction, the silk is pressed tightly down over the slit which is thus closed.

178. The Diving Bell.—A **Diving Bell** (fig. 201) is an apparatus for enabling a man to descend to a considerable depth under water to examine the foundation of a pier etc. It is a heavy, bell-shaped vessel of iron, closed at the top and open at the bottom and contains a platform inside. It is lowered by means of chains and sinks due to its own weight. As the bell descends, the pressure of water increases and compresses the air in the interior. Hence, to prevent the water from rising into the bell, and also to enable the workman to breathe, a constant supply of air is usually pumped into the bell through a tube from the surface by means of a condensing pump, the surplus air bubbling out from the lower edges of the bell.

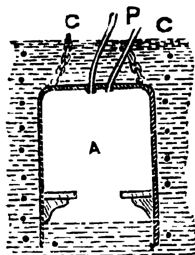


Fig. 201
A diving bell

It is easy to calculate how far the water will rise up inside the bell when it is immersed to a known depth, in such cases when no air is pumped into it from outside.

If the level of water inside the bell is at a depth h below the surface of water in which it is dipped, the pressure of the air enclosed within the bell A is

$$H + h$$

Where H is the height of the water barometer. Hence the volume of air which completely filled the bell at the atmospheric pressure H before immersion is now compressed to a smaller volume after immersion owing to the increased pressure $H + h$. It follows from Boyle's Law, that

$$\frac{\text{volume of air on immersion}}{\text{total volume of bell}} = \frac{H}{H + h}.$$

EXAMPLE :

A diving bell 10 ft. high and of uniform section is immersed down to the bed of a river 39 ft deep. How far will the water rise up inside the bell if the mercury barometer reads 30 inches ? (specific gravity of mercury = 13·6)

Let the water rise x ft. into the bell. Then pressure of air inside the bell is given by

p = atmospheric pressure + the pressure due to a column of water $(39 - x)$ ft. high

$$= \frac{30 \times 13{\cdot}6}{12} + (39 - x) \text{ ft. of water}$$

$$= (73 - x) \text{ ft.}$$

Hence

$$\frac{\text{volume of air in the bell}}{\text{volume of bell}} = \frac{\text{Atmospheric pressure}}{p}$$

$$\text{i.e.} \quad \frac{10 - x}{10} = \frac{34}{73 - x}$$

$$\text{or} \quad x^2 - 83x + 730 = 340$$

whence

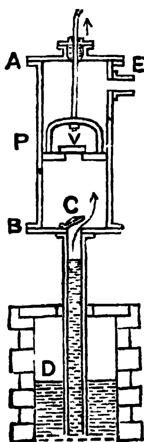
$$x = 5 \text{ ft.}$$

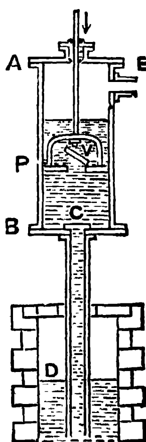
Hence the water will rise half way up the bell.

PUMPS

179. The Common Pump.—The **common pump**, also called the **suction pump** or the **lift pump** consists of a barrel or cylinder AB (fig. 202) in which works an airtight piston P. A long pipe BD is connected to the barrel at B and terminates beneath the surface of water which is to be raised. There are two valves, both opening upwards, one at V within the piston, closing an opening in it, and another at C the junction of the barrel and the pipe. The top of the barrel is generally provided with a spout E. In the case of a hand-pump, the piston rod is worked by means of a lever, often a bent one, called the pump-handle.

To explain the action, let us start with the piston at the bottom of the barrel and the pipe full of air





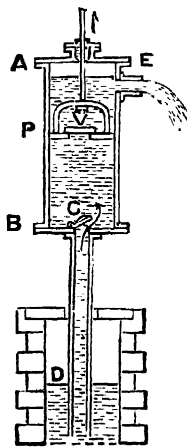


Fig. 202 Fig. 203 Fig. 204
The action of an water-pump

above the water surface. As the piston is raised (fig. 202), the space below the piston is increased

causing a fall of pressure in it : the atmospheric pressure acting on the valve V closes it ; the valve C, however, opens due to pressure of the air in pipe, which is greater than that of the air above the valve, and the air in BD expands into the part of the barrel below the piston. This causes the pressure on the surface of water in the pipe to be less than the atmospheric pressure which acting upon the water outside D forces it up in the pipe.

When the piston reaches the top, its motion is reversed. The pressure in the cylinder is increased, and closes the valve C. When the air below the piston is compressed to the atmospheric pressure, it begins to escape by pushing the valve V upwards (fig. 203). This continues till the piston is again at the bottom of the cylinder.

Other complete strokes follow, the water rising higher and higher in the cylinder until it begins to collect in the cylinder. When the piston is again lowered, water is forced through the valve V and at the next up-stroke of the piston flows out by the spout E (fig. 204).

Since water is raised in the tube solely by the atmospheric pressure, it follows that *the height of the piston above the surface of the water must never exceed the height of a water barometer* (*i. e.*, about 34 ft.). In practice, taking into account the weight of the valve etc., the limit of the working height of the piston is less than 34 ft.

180. The Force-Pump.—The **force pump** differs from the common pump in as much as the piston P is solid and has no valve : a pipe DE rises from a side close to the bottom of the cylinder and is provided with a valve D (fig. 205), opening outward from the cylinder.

At each down-stroke, the water collected in the

barrel is forced out through the valve D and up the delivery pipe E, and at each upstroke, the valve D

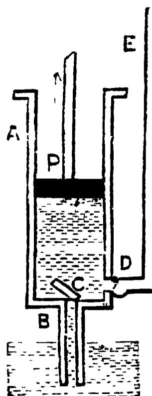


Fig. 205

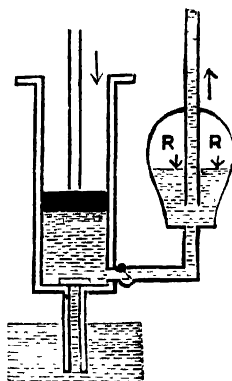


Fig. 206

The force-pump in action

closes due to the back-pressure of the water in the pipe E and water collects within the barrel.

The height to which the water can be forced depends on the force applied and the handle and the strength of the pump.

The drainage of deep mines is usually effected by a series of pumps. The water is first raised by one pump to a reservoir into which dips the suction tube of a second pump which sends the water up to a second reservoir, and so on. The piston rods of the different pumps are joined to a single rod, called the *spear*, which receives its motion from a steam-engine.

The flow in the delivery tube of the force-pump, as just described, will be intermittent, the water flowing only during the down-stroke of the piston. To obtain a continuous stream, two force-pumps may be so

joined as to have a common delivery tube and the two pistons in the two barrels worked by a common

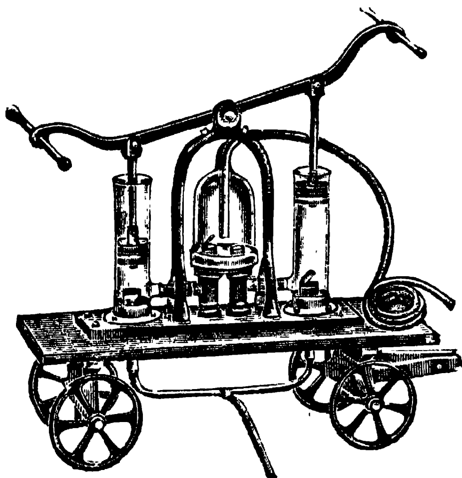


Fig. 207
A manual fire-engine

handle, so that as one piston descends, the other ascends. Even then the action of the pistons, momentarily stops when their motions are reversed.

In order to produce a *continuous* jet of water from the hose, an **air-chamber** is provided with the pump. This is simply a large metal dome (R, in fig. 206) partly filled with air. The delivery tube leads into this chamber whence a hose E, one end of which is near to the bottom of the dome, leads up to the height required. When the water is rapidly pumped into the chamber, it rises above the lower open end of the hose, and compresses the air in the chamber, while part of the water is forced out at the same

time. During the upward stroke of the piston the valve T is closed and the air in the chamber being no longer subjected to the pressure due to the piston expands, thus driving the water up the hose. Thus a continuous flow of water is maintained.

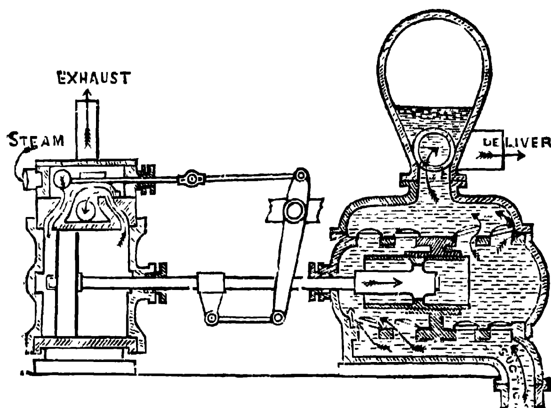


Fig. 208
Steam fire-engine*

Fire-Engines.—The **fire-engine** is a contrivance by which water is pumped up from a tank or reservoir and projected under high pressure to extinguish fire. When the engine is worked by muscular power it is called a manual fire-engine. When steam is used to work the engine, it is called a steam fire-engine.

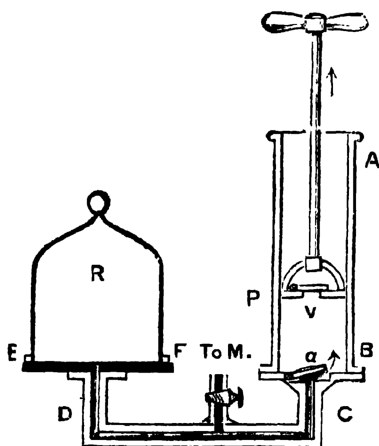
The **Manual Fire-engine** (fig. 207) consists of two force-pumps connected to a common air-chamber. The **Steam Fire-engine** (fig. 208) is a double-action force-pump with a horizontal barrel. The piston is driven backwards and forwards by steam-power, and water enters the barrel on the two sides of the

* The figure is adapted from a drawing given in the First Course of Physics by Millikan and Gale.

piston alternately. Each half forms a complete pump. The student will find it quite interesting to follow the action of the pump from a study of the diagram given.

181. Mechanical Air-Pumps.—The air-pump is an instrument constructed for the purpose of pumping air out of a closed vessel. Air-pumps may be of two distinctly different types: one type being known as the *mechanical air-pump*, and the other as the *mercury pump*.

The air pump was invented by OTTO VON GUERICKE,



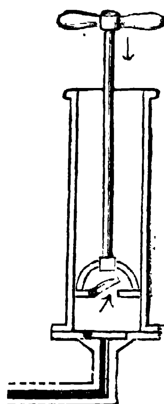


Fig. 209 Fig. 210

An air-pump in action

in about 1650. It is almost identical in construction and similar in action with the common water-pump described in art. 180. This consists of a cylindrical metal barrel AB (fig. 209), in which an air-tight

piston, P, can be worked up and down by a handle. This piston has a valve opening upwards. The barrel communicates through the tube D with the bell-jar receiver or a vessel R, to be exhausted, which fits air tight on the flat circular disc EF. At the junction of the barrel and the tube D there is a second valve a, also opening upwards. D is provided with a stop-cock (not shown in the figure), by turning which air may be admitted into the receiver. To indicate the exent of exhaustion in R, a manometer may be connected with the pipe D by a brass side-tube.

To understand the action, suppose the piston to be at the bottom of the barrel. In the up-stroke, the valve in the piston at once closes and the pressure of air within the barrel below P falls. The air in the receiver and the tube lifts the valve a, and expands into the barrel; thus the pressure in R is reduced. When the piston is forced downwards (fig. 210), it compresses the air in the barrel below it. This at once closes the valve a; when the pressure of the air in the barrel becomes greater than the atmospheric pressure, the piston valve opens and allows the air to escape from beneath the piston to the upper part of the barrel and thence to the outer air. At the next upstorke, the air left in the receiver will again expand so as to fill both the receiver and the barrel. This process of exhaustion goes on until a fairly low vacuum is produced in the receiver.

To calculate the density of the air left in the receiver after a number of strokes is rather simple. Let

V = volume of the receiver R and pipe D.

v = volume of the barrel between the two valves when the piston is at the top of its upstroke.

It is clear that at each upstroke a volume V of air expands

to $V+v$, and thereby becomes rarefied. If the oirginal density of air be D, the density d_1 after the first upstroke is given by

$$VD=(V+v).d_1.$$

Or $$d_1=\frac{V}{V+v}.D$$

Similarly, the density d_2 after the second upstroke,

$$d_2 = \frac{V}{V+v}.d_1=\left(\frac{V}{V+v}\right)^2.D$$

Thus the density of the air left in the receiver after n upstrokes is given by

$$d_n=\left(\frac{V}{V+v}\right)^n.D \qquad \ldots \qquad (101)$$

According to Boyle's Law, the pressure of a gas is proportional to its density; we have, therefore,

$$p_n=\left(\frac{V}{V+v}\right)^n.P$$

where Pn is the pressure after n upstrokes and P is the original pressure of the air in the vessel.

The result shows that the value of d_n can never become zero (indicating a perfect vacuum), but it may be made very small after a sufficient number of strokes, provided the pump is mechanically perfect.

But a pump of this pattern is never mechanically perfect. There is always a certain amount of leakage in action. Again there must always be a small *clearance i.e*, a space left at the bottom of the barrel even when the piston is pushed 'full home.' After pumping for sometime there comes a stage when the valves do not open, and the air between them alternately expands into the barrel and is forced back into the clearance. Further, the valves, however light, require a force to be opened, and when the pressure of the air in the receiver becomes very low, it is unable to raise the valve *a* during the upstroke of the piston. Another point to be noticed in the working of the pump is that when the exhaustion is carried to a certain extent, the excess of pressure on the upper side of the piston over that of the air in the barrel below it makes the pump hard to work.

182. The Double-barrelled Pump.—In the ordinary single-barrelled pump no air is expelled in the downstroke ; in the DOUBLE-BARRELLED PUMP, also called *Hawksbee's Air-Pump*, there are two barrels instead of one, and the pistons are worked up and down by means of a *rack-and-pinion* arrangement so that when the pinion is turned by a lever handle, one piston rises while the other falls ; thus air is thrown out during each storke. It should be noted that the passages from the two barrels unite into a single passage.

This arrangement possesses two advantages. First, the air is taken out twice as quickly as with a single barrel. Secondly, since the atmospheric pressure tends to depress each piston, its effect on one of the pistons which is rising, is just balanced by that on the other which is descending ; a less force, therefore, is necessary to raise the piston than that in the ordinary pump.

Uses of the Air-pump :—At the time when the air-pump was invented, experiments were devised to demonstrate the effect of a vacuum, some of which have already been described.

Besides its use in the laboratory, the exhausting air-pump finds an application in many industries. It is employed in sugar refinery to lower the boiling point of the syrup ; in exhausting the globes of incandescent electric lamp ; in parts of ice-making machinery ; in exhausting the air from vessels meant for preserving foods etc., etc.

183. The Condensing Pump.—This is an air-pump for compressing the air. It consists of a barrel AB, in which works a piston P (Fig. 211). AB communicates at one end through a stop-cock with the Receiver or the vessel into which air is to be

compressed. Both the piston and the end of the barrel contain valves E and F, opening towards the receiver.

Let the piston be at the end of the barrel near the valve F. In the *backward* stroke, the pressure in the barrel below the piston is reduced ; the valve F is closed by the pressure in the receiver while the atmospheric pressure opens the valve E, and the barrel is filled with air at the atmospheric pressure. In the *forward* stroke, the valve E is closed and F is opened ; hence all the air from the barrel is forced into the receiver. This process is repeated in every complete stroke of the piston.

The piston valve is not necessary, if there be a hole in the side of the barrel just below the outermost position of the piston.

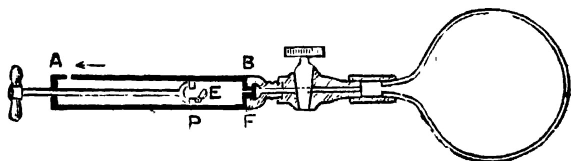


Fig. 211
A condensing pump

The pressure of the compressed air after a given number of strokes of the piston may be easily calculated. Let

V = volume of the receiver
v = volume of the barrel
D = density of the atmospheric air

At each backward stroke, a volume v of air at the atmospheric pressure enters the barrel. At each forward stroke this air enters the receiver.

Hence after n *complete* strokes the mass of air in the receiver $= (V+nv).\ D$

But its actual volume is V. Let its raised density be d_n. Then $V.\ d_n = (V+nv).\ D$

$$\therefore \quad d_n = (1+nv/V).\ D \qquad \ldots \qquad (102)$$

If Boyle's law is assumed to hold here, the pressure within R after n strokes is given by

$$pn = \pi\,(1+nv/V)$$

where π is the atmospheric pressure.

In the common *Bicycle Pump* or *Cycle Tyre Inflator* (Fig. 212), the valve in the piston is replaced by a contrivance called the **Cup-valve.** A cup-shaped disc of leather, a little larger than the barrel of the pump is attached to a loosely fitting metal piston composed of two circular plates of smaller diameter than that of the barrel, between which the leather is held. During the up-stroke the cup collapses inwards and allows the air to pass by it; on the down-stroke the leather presses tight against the walls of the barrel preventing the escape of air round the piston.

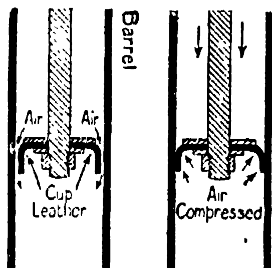


Fig 212
Bicycle pump

184. The Toepler Pump.—The amount of vacuum produced by the types of pump just described is limited by the inefficiency of the valves and other mechanical drawbacks. To produce very high vacuum, therefore, we have to take recourse to other types of pump, two of which we shall describe here.

In its simplest form the Toepler pump consists of a glass bulb A fitted with two barometer tubes BC and DE as shown in fig. 213. The open end of BC dips into a cistern of mercury and that of DE is connected by means of a rubber tubing to a mercury reservoir G. A side tube F fixed to DE at the junc-

tion D connects the apparatus to the vessel to be evacuated.

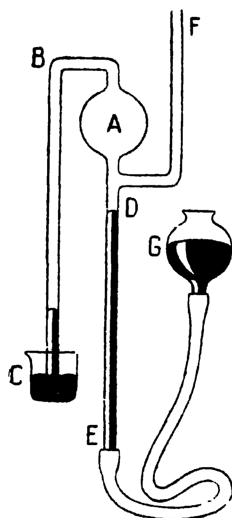


Fig. 213
Toepler pump

To work the pump the reservoir G is raised ; mercury rises up into DE and eventually seals off the junction D. The communication with the side tube is then cut off. On still raising G, mercury fills the bulb A driving away all the air inside it through BC. The air escapes bubbling through the mercury in the cistern. G is then lowered, the mercury sinks down in DE opening the junction D. The vacuum produced in A sucks in some of the gas from the vessel to be exhausted. G is then raised again and the entrapped gas is forced out through BC as before. This process is repeated until the vessel is exhausted to the desired extent.

It may be noted here that the action of the Toepler pump is very slow but the vacuum it can produce is of the order of the vapour pressure of mercury at the room temperature.

185. The "Hyvac" Pump.—The "Hyvac" pump is one of the best pumps used in modern times to produce very high vacuum such as those in X-Ray tubes, discharge tubes, thermionic valves, etc. It is shown in sections in fig. 214

C is a cylindrical casing in which rotates an

eccentric rotor R (Fig. 214) mounted on a shaft along the axis of the cylinder. The inlet tube E communicates with the vessel to be exhausted and the valve at O serves as an exhaust valve. F is a rigid fibre block pressed air-tight by means of a spring (not shown in the diagram) on to the surface of the rotor.

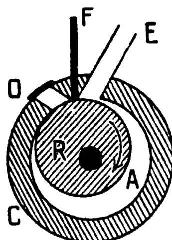


Fig. 214
Hyvac pump

In the position shown in figure, the space A is in communication with the vessel to be evacuated. As the rotor revolves in the direction shown by the arrow, the space A is gradually diminished thus compressing the gas enclosed in it. During a complete revolution, there is one position of the rotor for which the volume of the entrapped gas is a minimum and the compression is so great that the exhaust valve O opens and the gas rushes out. This process is repeated in each revolution and as the rotor is rapidly rotated, the vessel can be evacuated within a few minutes.

Exercise XX

1. Describe a suction pump. Water cannot be raised to a height much greater than 30 ft. by means of such a pump. State the reason for this, and describe a laboratory experiment by which you could prove your explanation to be correct.

[C. U.—1930]

2. Explain the action of a siphon.

A siphon is used to empty a cylindrical vessel filled with mercury. The shorter limb of the siphon reaches to the bottom of the vessel, which is 45 inches deep, but it is found that the mercury ceases to run before the vessel is empty. Explain this. [C. U.—1926]

3. Explain the principle and action of the siphon, and state how the principle is used in Tantalus' cup. [C. U.—1928]

4. Describe an air-pump and explain its action. [C. U.—1933]

5. If the receiver of an exhaustion pump has double the volume of the barrel, find the density of the air remaining after 10 strokes, neglecting leakage etc.

6. Air is forced into a vessel by a compression-pump whose barrel has 1/10 of the volume of the vessel. Compute the density of the air in the vessel after 20 strokes.

7. Describe in detail, with a diagram, a condensing pump and its mode of action.

The barrel and receiver of a condensing pump have capacities of 75 c.c. and 1,000 c.c. respectively. How many strokes will be required to raise the pressure of the air in the receiver from one to four atmospheres? [C. U.—1925]

8. Describe and explain the principle of action of a force pump. [C. U.—1937]

9. Describe a double barrelled air-pump and explain its action.

A lift pum is used to pump oil of sp. gr. 0.8 from a lower into an upper tank. What is the maximum possible height of the pump above the lower tank when the atmospheric pressure is 76 cms of mercury. Give reasons for your answer. [C. U.—1938]

ANSWERS.

Exercise I

2. (*b*) 1 ft. = 30·48 cm.
   1 in. = 2·54 cm.

4. 1 kg. = 2·204 lbs.
   1 gm. = 0·002 lb.

5. About 64·05 gms.

9. 1 radium = 57°·296.

Exercise II

5. 8 min. 39·6 secs. ; 120° with the direction of the current.

6. $\frac{11}{75}$ ft. per sec^2.

9. 4·24 miles per hour.

10. 46·9 ft. per sec.

Exercise III

1. 25 lbs. 2. 3·46 lbs

3. Force $\frac{1}{\text{distance}}$.

4. 3ft. from the man bearing 94 lbs.

5. 32 lbs.

Exercise IV

5. 1 poundal = 13825·7 dynes.

6. 25 gms. 7. 80 poundals.

8. 5475·6 poundals.

9. 180 poundals,

10. 45 .

Exercise V

1. 176 ft. ; 576 ft.

3. 350 ft.

4. 4·4 ft. ; 0·16 ft.

5. 2·004 secs.

6. 6·5 secs.

8. $29\frac{1}{3}$ ft. per sec.

9. 1200 ft. per sec.

11. 88 ft. per sec. , 24 ft. per sec.

Exercise VI

4. 20 lbs.

5. $14\frac{1}{21}$ in. from the end of the tube weighing 8 oz.

6. $h = \frac{1}{\sqrt{10}}$ ft.

7. $2\frac{6}{7}$ ft. from the end nearest to 1 lb.

8. $\frac{5}{16}$ from the heavier end.

9. 0·0721 in. from centre of disc.

Exercise VII

3. 120 gms. wt.

4. 0·75. 5. 3·46 lbs.

Exercise VIII

1. 3 lbs.

2. 22 8′ with the horizon.

3. 12 in. from 10 lbs. wt.

4. 50 lbs. 5. 840 lbs.

6. $373\frac{1}{3}$ lbs. 7. 21 lbs.

Exercise IX

1. 41. 3. 09·39 cms.

5. 9·3 cms.

6. 99·39 cms. ; loses.

10. 0·004 in.

Exercise X

10. $3·84 \times 10^{11}$ ft. lbs.

13. $2·17 \times 10^{9}$ ft. poun-

dals ; $3·77 \times 10^{8}$ ft. poundals.

Exercise XI

2. 8·85 c.c.
3. 5 gms. per c.c.

Exercise XII

2. 1 mm.
5. $2·4 \times 10^{12}$ dynes per cm^2.

Exercise XIII

3. 1.11×10^{6} dynes per cm^2.
5. $6·67 \times 10^{5}$ dynes per cm^2.
6. 1000 gms. wt. (top) ; 2000 gms. wt. (bottom) ; 1500 gms. wt. (sides).
9. 640·6 lbs. per sq. ft.
10. 10·19 metres nearly.

Exercise XV

2. 3·17 gms. per c.c. ; 0·5 gms. per c.c.
3. 20 c.c.
5. 36 c.c. ; 7·6 gms. per c.c. approximately.
6. 2·37 gms. per c.c.
8. 30 gms. 9. 360 gms.
10. 13·92 gms.
11. $\frac{11}{38}$. 12. 8 ins.

Exercise XVI

1. 50 gms. to be added.
2. 82 gms.
3. 180 gms. 4. 70·8.
6. 6437·5 cu. yds.
7. 21. 8. 0·25. 10. 4·0.
11. 0·89 c.c. 12. 0·795.
13. Floats. 14. 1·2.

Exercise XVIII

5. 8264·46 litres.
6. $1·012 \times 10^{6}$ dynes per cm^2.
7. 15·7 lbs. per sq. in.
12. $1·014 \times 10^{6}$ dynes per cm^2.
13. 25·2 ft. approximately.

Exercise XIX

1. 47·6 metres.
2. $1·4 \times 10^{6}$ dynes per cm^2.
3. 0·131 c.c.
4. 11·31 litres nearly.
5. 1·327 gms. approximately.
6. 500 mm.
8. 6 times the initial pressure. 11. 2 ins.
12. 12 ins. 13. 70 cms.
14. 0·068 of the original volume.
15. 1·012 dynes per cm^2 ; 0·5 gm. nearly.

Exercise XX

5. $1·73 \times 10^{-2}$ of the original density.
6. 3 times the original density.
7. 40 Strokes.
9. 12·92 metres.

INDEX

THE REFERENCES ARE TO PAGES

ERRATA

Page 109, Exercise IV, **Q. 8**, the answer is 5475·6 poundal

Page 109, Exercise IV, **Q. 10**, Read 72 ft. in place of [illegible]0 ft.

Page 129, Exercise V, **Q. 9**, Read 2[illegible] in place of $\frac{27}{12}$

Page 199, Exercise IX, **Q. 3**, Insert "Find the length of a simple seconds pendulum" before "at a place where g is 981."